On the behavior at infinity of one class of homeomorphisms

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Let Γ be a family of curves γ in $\mathbb{R}^n$, $n \geqslant 2$. A Borel measurable function $\rho : \mathbb{R}^n \to [0, \infty]$ is called *admissible* for Γ, (abbr. $\rho \in \mathrm{adm}\,\Gamma$), if

$$\int\limits_{\gamma} \rho(x)\, ds \;\geqslant\; 1$$

for any curve $\gamma \in \Gamma$. Let $p \in (1, \infty)$. The quantity

$$M_p(\Gamma) \;=\; \inf_{\rho \in \mathrm{adm}\,\Gamma} \int\limits_{\mathbb{R}^n} \rho^p(x)\, dm(x)$$

is called *p–modulus* of the family Γ.

For arbitrary sets E, F and G of $\mathbb{R}^n$ we denote by $\Delta(E, F, G)$ a set of all continuous curves $\gamma : [a, b] \to \mathbb{R}^n$ that connect E and F in G, i.e., such that $\gamma(a) \in E$, $\gamma(b) \in F$ and $\gamma(t) \in G$ for $a < t < b$.

Let D be a domain in $\mathbb{R}^n$, $n \geqslant 2$, $x_0 \in D$ and $d_0 = \mathrm{dist}(x_0, \partial D)$. Set

$$\mathbb{A}(x_0, r_1, r_2) = \{x \in \mathbb{R}^n : r_1 < |x - x_0| < r_2\}\,,$$

$$S_i = S(x_0, r_i) = \{x \in \mathbb{R}^n : \, |x - x_0| = r_i\}\,, \quad i = 1,\, 2\,.$$

Let a function $Q : D \to [0, \infty]$ be Lebesgue measurable. We say that a homeomorphism $f : D \to \mathbb{R}^n$ is ring Q-homeomorphism with respect to p-modulus at $x_0 \in D$ if the relation

$$M_p(\Delta(fS_1, fS_2, fD)) \;\leqslant\; \int\limits_{\mathbb{A}} Q(x)\, \eta^p(|x - x_0|)\, dm(x)$$

holds for any ring $\mathbb{A} = \mathbb{A}(x_0, r_1, r_2)$, $0 < r_1 < r_2 < d_0$, $d_0 = \mathrm{dist}(x_0, \partial D)$ and for any measurable function $\eta : (r_1, r_2) \to [0, \infty]$ such that

$$\int\limits_{r_1}^{r_2} \eta(r)\, dr = 1\,.$$

Denote by ω_{n-1} the area of the unit sphere

$$\mathbb{S}^{n-1} = \{x \in \mathbb{R}^n : \, |x| = 1\}$$

in $\mathbb{R}^n$ and by

$$q_{x_0}(r) = \frac{1}{\omega_{n-1}\, r^{n-1}} \int\limits_{S(x_0, r)} Q(x)\, d\mathcal{A}$$

the integral mean over the sphere

$$S(x_0, r) = \{x \in \mathbb{R}^n : \, |x - x_0| = r\},$$

where $d\mathcal{A}$ is the element of the surface area. Let $L(x_0, f, R) = \sup\limits_{|x - x_0| \leqslant R} |f(x) - f(x_0)|\,.$

Theorem 1. *Suppose that $f:\mathbb{R}^n\to\mathbb{R}^n$ is a ring Q-homeomorphism with respect to p-modulus at a point x_0 with $p>n$ where x_0 is some point in $\mathbb{R}^n$ and for some numbers $r_0>0$, $K>0$ the condition*

$$q_{x_0}(t)\leqslant K\,t^{\alpha}$$

holds for a.e. $t\in[r_0,+\infty)$. If $\alpha\in[0,p-n)$ then

$$\varliminf_{R\to\infty}\frac{L(x_0,f,R)}{R^{\frac{p-n-\alpha}{p-n}}}\geqslant K^{\frac{1}{n-p}}\left(\frac{p-n}{p-n-\alpha}\right)^{\frac{p-1}{p-n}}>0\,.$$

If $\alpha=p-n$ then

$$\varliminf_{R\to\infty}\frac{L(x_0,f,R)}{(\ln R)^{\frac{p-1}{p-n}}}\geqslant K^{\frac{1}{n-p}}\left(\frac{p-n}{p-1}\right)^{\frac{p-1}{p-n}}>0\,.$$

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