Fejer Sums and the von Neumann Ergodic Theorem

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The Fejér sums of periodic measures and the norms of the deviations from the limit in the von Neumann ergodic theorem are calculated, in fact, using the same formulas (by integrating the Fejér kernels), so this ergodic theorem is a statement about the asymptotics of the growth of the Fejér sums at zero for the spectral measure of the corresponding dynamical system.

As a result, well-known estimates for the rates of convergence in the von Neumann ergodic theorem can be restated as estimates of the Fejér sums at the point for periodic measures. For example, natural criteria for the polynomial growth and polynomial decrease in these sums can be obtained.

On the contrary, available in the literature, numerous estimates for the deviations of Fejér sums at a point can be used to obtain new estimates for the rate of convergence in this ergodic theorem. For example, for many dynamical systems popular in applications, the rates of convergence in the von Neumann ergodic theorem can be estimated with a sharp leading coefficient of the asymptotic by applying S.N. Bernstein's more than hundred-year old results in harmonic analysis.

References

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