Geodesics on regular tetrahedra in spherical space

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The full classification of closed geodesics on regular tetrahedra in Euclidean space follows from the tiling of Euclidean plane by regular triangles [1].

In [2] we described a complete classification of simple (without self-intersections) closed geodesics on regular tetrahedra in hyperbolic 3-space. In addition, we presented the asymptotic of the number of simple closed geodesics of length not greater than L as L tends to infinity.

In current work we described all simple closed geodesics on regular tetrahedra in three-dimensional spherical spaces. In this space the tetrahedron's curvature is concentrated both into its vertices and into its faces. The value α of the faces' angle of a tetrahedron satisfies $\pi/3 < \alpha < 2\pi/3$. The intrinsic geometry of a tetrahedron depends on α.

By definition a simple closed geodesic on a tetrahedron has the type (p, q) if it has p points on each of two opposite edges of the tetrahedron, q points on each of another two opposite edges, and there are $(p + q)$ points on each edges of the third pair of opposite one.

We proved that on a regular tetrahedron in spherical space there exists the finite number of simple closed geodesic. The length of all these geodesics is less than 2π. Furthermore, for any coprime integers (p, q) we found the numbers α_1 and α_2 depending on p, q and satisfying the inequalities $\pi/3 < \alpha_1 < \alpha_2 < 2\pi/3$ such that

1) if the faces' angle of a regular tetrahedron is measured $\alpha \in (\pi/3, \alpha_1)$ then in spherical space there exists unique, up to the rigid motion, simple closed geodesic of type (p, q) on this tetrahedron,

2) if the value α of faces' angle is in $(\alpha_2, 2\pi/3)$ then there is no simple closed geodesic of type (p, q) on the tetrahedron with such faces' angle in spherical space.

References

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[2] A. A. Borisenko, D. D. Sukhorebska. Simple closed geodesics on regular tetrahedra in hyperbolic space, *Mat. Sb.*, 211:5, 2020, p. 3-30 (in Russian). *English translation*: SB MATH, 2020, 211(5), DOI:10.1070/SM9212