Foliations of 3-manifolds with small module of mean curvature

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A taut foliation is a codimension one transversely oriented foliation of an oriented 3-manifold M with the property that for each leaf there is a transverse circle intersecting it.

D. Sullivan proved that $\mathcal{F}$ is taut iff each leaf is a minimal surface for some Riemannian metric on M which is equivalent that $\mathcal{F}$ does not contain generalized Reeb components [1]. Recall that a surface F is called minimal if the mean curvature H of F is identically zero.

In the present work we announce the following result.

Theorem 1 (Main theorem). *Let (M, g) be a closed oriented Riemannian 3-Manifold sutisfying the following properties:*

(1) *the volume $Vol(M, g) \le V_0$;*
(2) *the sectional curvature γ of (M, g) satisfies $\gamma \le \gamma_0$ for the constant $\gamma_0 \ge 0$;*
(3) $\min\{inj(M, g), \frac{\pi}{2\sqrt{\gamma_0}}\} \ge i_0$.

Then there is a constant $H_0(V_0, \gamma_0, i_0)$ such that any transversally oriented foliation $\mathcal{F}$ of codimension one on M with the mean curvature of the leaves satisfying $|H| < H_0$, must be taut. The constant H_0 is defined as wollowing:

$$H_0 = \begin{cases} \min\{\frac{2\sqrt{3}i_0^2}{V_0}, \sqrt[3]{\frac{2\sqrt{3}}{V_0}}\}, & \text{if } \gamma_0 = 0, \\ \min\{\frac{2\sqrt{3}i_0^2}{V_0}, x_0\}, & \text{if } \gamma_0 > 0, \end{cases} \tag{$*$}$$

where x_0 is the positive root of the equation

$$\frac{4}{\gamma_0} \operatorname{arcctg}^2 \frac{x}{\sqrt{\gamma_0}} - \frac{2V_0}{\sqrt{3}} x = 0.$$

Let us recall that, by the Novikov's theorem [2], if $\pi_1(M) < \infty$ or $\pi_2(M) \neq 0$, then excepting the foliation of $S^2 \times S^1$ by spheres, $\mathcal{F}$ contains a Reeb component. Thus we obtain the following corollary.

Corollary 2. *Let (M, g) be a Riemannian manifold that satisfies the properties in the theorem above. If $\pi_1(M) < \infty$ or $\pi_2(M) \neq 0$, then excepting the foliation of $S^2 \times S^1$ by spheres, (M, g) does not admit a foliation with the mean curvature H of leaves satisfying the inequality $|H| < H_0$, where H_0 is determined from $(*)$.*

References

[1] Dennis Sullivan. A homological characterization of foliations consisting of minimal surfaces *Commentari Math. Helvetici* , 54 : 218–223, 1979.
[2] С. П. Новиков. Топология слоений *Тр. Моск. мат. о-ва.*, 14 : 249–278, 1965.