Gromov-Witten invariants and identification of the energy levels of solitonic states

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Calabi-Yau manifold can be represented in terms of toric data [1]. Such presentation allows us to go to the dual polyhedron, through which are defined the gauge groups. As an example, we can consider Calabi-Yau manifold $X_{24}(1,1,2,8,12)^{3,243}_{-480}$ determined by Gromov-Witten invariants [2], presented in table 1. Calabi-Yau manifold is defined by n - Gromov-Witten invariants, the analogue of the principal quantum number in physics and by (a, b, c) - the internal quantum numbers.

Table 1

Gromov-Witten invariants for $X_{24}(1,1,2,8,12)^{3,243}_{-480}$

a, b, c	n	a, b, c	n	a, b, c	n	a, b, c	n
(0,0,1)	-1	(0,1,1)	-1	(0,1,2)	-2	(0,1,3)	-3
(0,1,4)	-4	(0,1,5)	-5	(0,2,3)	-3	(0,2,4)	-16
(1,0,0)	240	(1,0,1)	240	(1,1,1)	240	(1,1,2)	720
(1,1,3)	1200	(1,1,4)	1680	(1,2,3)	1200	(2,0,0)	240
(2,0,2)	240	(2,2,2)	240	(3,0,0)	240	(3,0,3)	240
(4,0,0)	240	(5,0,0)	240	(6,0,0)	240	(0,1,0)	0

An embedding of Gromov-Witten invariants is presented by the formula

$$n_{a,b,c} = \sum_d n_{a,b,c,d} \ . \tag{1}$$

Such an embedding suggests the possibility of a phase transition between different manifolds characterized by different n. In [3], for the case of an elliptic fibration representing Calabi-Yau manifolds, the matter content of the charged fields of the effective theory is associated with divisors of the base of the fibration. The representations of groups of these fields is determined by the of intersection number of these divisors, in other words, gauge groups are associated with curves of singularities of the manifold and intersecting singularity curves define charged fields classified by gauge groups. Table 2 from [3] represents such a set of fields and the corresponding groups. From table 2 it can be seen that matter content presented by gauge groups can be embeded according to formula (0.1) through transitions between gauge groups

$$E_7 \to E_6 \to SU(6) \to SU(5) \to SU(4) \to SU(3) \to SU(2) \ ,$$

characterized by different sets of the matter content of the charged fields.

The presence of Gromov-Witten invariants, n signals about the presence of a central charge, Z or mass, M of a solitonic object, [4]

$$e^{\int_\eta S} = e^{2\pi i \tau \eta}, \ \eta = n\varphi, \ M = Z = \int_D \Omega = n_i \varphi_i$$

Table 2

Matter content of Calabi-Yau models, characterized by gauge groups (for n=2,4,6,8,10,12)

Group	Matter content
$SU(2)$	$(6n+16)\mathbf{2}$
$SU(3)$	$(6n+18)\mathbf{3}$
$SU(4)$	$(n+2)\mathbf{6}+(4n+16)\mathbf{4}$
$SU(5)$	$(3n+16)\mathbf{5}+(2+n)\mathbf{10}$
$SO(10)$	$(n+4)\mathbf{16}+(n+6)\mathbf{10}$
E_6	$(n+6)\mathbf{27}$
E_7	$\left(\frac{n}{2}+4\right)\mathbf{56}$

Here the holomorphic 3-form Ω is defined on Calabi-Yau X, (D is a cycle in X). Gromov-Witten invariants, n_i are connected with the cohomology classes, φ_i of rational curves in X into central charge, Z.

Solitonic objects of instanton type are characterized by the condition on the action, S

$$S \geq \left(\frac{8\pi^2}{g^2}\right)|Q| = E$$

where

$$Q = -\frac{1}{16\pi^2}\int d^4xTr[\tilde{F_{\mu\nu}}F_{\mu\nu}]$$

Pontryagin's homotopy index, defined by the Yang-Mills field strength, $F_{\mu\nu}$, [5]. Consequently, the central charge has an interpretation of a multiple topological quantum number or the number of the Bogomolny-Prasad-Sommerfeld, corresponding to the multiplicities of degeneration of different configurations of solitonic type in Calabi-Yau space.

References

[1] A. Klemm et al. Calabi-Yau fourfolds for M- and F-theory compactifications. *Nucl. Phys.*, B518: 515–574, 1998.
[2] M. Bershadsky et al. Geometric singularities and enhanced gauge symmetries. *Nucl. Phys.*, B481: 215–252, 1996.
[3] P. Candelas, E. Perevalov, G. Rajesh. Matter from toric geometry. *Nucl. Phys.*, B519: 225-238, 1998.
[4] David R. Morrison. Gromov-Witten invariants and localization, arXiv:1608.02956 [hep-th].
[5] R. Rajaraman. An Introduction to Solitons and Instantons in Quantum Field Theory. North Holland, 418p, 1987.