On a regularized solution of the Cauchy problem for matrix factorizations of the Helmholtz equation in $m-$dimensional bounded domain

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In the paper it is considered the problem of regularization of the Cauchy problem for matrix factorisations of the Helmholtz equation in $m-$dimensional bounded domain of the type of a curvilinear triangle. Using the results of works [1]-[2], is constructed explicitly Carleman matrix and, based on the regularized solution of the Cauchy problem.

Let $\mathbb{R}^m$ be a $m-$dimensional real Euclidean space,

$$x = (x_1, \dots, x_m) \in \mathbb{R}^m, \ \ y = (y_1, \dots, y_m) \in \mathbb{R}^m.$$

Let $G_\rho \subset \mathbb{R}^m$ be a bounded simply connected domain whose boundary consists of the surface of a cone

$$\left|y'\right| = \tau y_m, \ \tau = tg\frac{\pi}{2\rho}, \ y_m > 0, \ \rho > 1,$$

and a smooth piece of the surface S lying inside the cone, i.e. $\partial G_\rho = S \bigcup T, \ T = \partial G_\rho \backslash S$.

We consider in the domain G_ρ a system of differential equations

$$D\left(\frac{\partial}{\partial x}\right) U(x) = 0, \tag{1}$$

where $D\left(\frac{\partial}{\partial x}\right)$ is the matrix of differential operators is of the first order.

We denote by $A(G_\rho)$ the class of vector functions in the domain G, of continuous on $\overline{G}_\rho = G_\rho \bigcup \partial G_\rho$ and satisfying system (1).

Problem 1. Let $U(y) \in A(G_\rho)$ and

$$U(y)|_S = f(y), \ y \in S. \tag{2}$$

Here, $f(y)$ is a given continuous vector function on S.

It is required to restore the vector function $U(y)$ in the domain G_ρ, based on its values $f(y)$ on S.

Theorem 2. *Let $U(y) \in A(G_\rho)$ on the entire boundary of ∂G_ρ satisfy the boundary condition*

$$|U(y)| \leq 1, \ y \in T.$$

Then we have the estimate

$$\left|U(x) - U_{\sigma(\delta)}(x)\right| \leq C_\rho(\lambda, x)\sigma\delta^{\left(\frac{\gamma}{R}\right)^\rho}, \ \sigma > 1, \ x \in G_\rho.$$

Corollary 3. *The limiting equality*

$$\lim_{\delta \to 0} U_{\sigma(\delta)}(x) = U(x),$$

holds uniformly on each compact set in the domain G_ρ.

Thus, the functional $U_{\sigma(\delta)}(x)$ determines the regularization of the solution of problem (1)-(2).

References

[1] Juraev D.A. On the Cauchy problem for matrix factorizations of the Helmholtz equation in a bounded domain. *Sib. Electron. Mat. Izv.*, 15 : 11–20, 2018.

[2] Juraev D.A. On a regularized solution of the Cauchy problem for matrix factorizations of the Helmholtz equation. *Advanced Mathematical Models & Applications*, 4(1) : 86–96, 2019.