Extensions of almost orthosymmetric lattice bimorphisms

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Let E ,F and G be vector lattices. We say that a linear operator $T : E \to F$ is a lattice homomorphism if $T(x \vee y) = Tx \vee Ty$ for every $x, y \in E$. A bilinear map $\Phi : E \times F \to G$ is said to be positive if $|\Phi(x, y)| \leq \Phi(|x|, |y|)$ for all $x \in E$ and $y \in F$. The bilinear map $\Phi : E \times F \to G$ is said to be lattice bilinear map (or lattice bimorphism) whenever it is separately lattice homomorphisms for each variable or equivalently, $|\Phi(x, y)| = \Phi(|x|, |y|)$ for all $x \in E$ and $y \in F$. Let E and F be Archimedean vector lattices. A bilinear map $T : E \times E \to F$ is called an orthosymmetric if $x \wedge y = 0$ implies $T(x, y) = 0$ for all $x, y \in E$. A vector lattice E is called Dedekind complete if every non-empty subset of E which is bounded from above has a supremum. A Dedekind complete vector lattice M is said to be a Dedekind completion of the vector lattice E whenever E is Riesz isomorphic to a majorizing order dense Riesz subspace of M. Denote by E^δ the Dedekind completion of E. Every Archimedean vector lattice has a unique Dedekind completion. A vector lattice E is said to be universally complete if E is Dedekind complete and every pairwise disjoint positive vectors in E has a supremum in E. Every Archimedean vector lattice E has a universal completion E^u. It means that there exists a unique (up to a lattice isomorphism) universally complete vector lattice E^u such that E is Riesz isomorphic to an order dense Riesz subspace of E^u.

Definition 1. Let E and F be Archimedean vector lattices. A bilinear map $T : E \times E \to F$ is called an almost orthosymmetric if $x \wedge y = 0$ implies $T(x, y) \wedge T(y, x) = 0$ for all $x, y \in E$, [13].

Every orthosymmetric bilinear map is an almost orthosymmetric, but the converse is not always true.

Let E be an Archimedean vector lattice and F be a Dedekind complete vector lattice.

In this talk, we show that if $T : E \times E \to F$ is an almost orthosymmetric lattice bimorphism, then extension of T , $T^\sim : E^\delta \times E^\delta \to F$, is an almost orthosymmetric lattice bimorphism.

Theorem 2. *Let E be an Archimedean vector lattice and let E^δ be its Dedekind completion and let F be a Dedekind complete vector lattice. If $\Phi : E \times E \to F$ is an almost orthosymmetric lattice bimorphism, then Φ can be extended to an almost orthosymmetric lattice bimorphism $\Psi : E^\delta \times E^\delta \to F$.*

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