The commutator and centralizer of Sylow subgroups of alternating and symmetric groups, its minimal generating set

Ruslan Viacheslavovich Skuratovskii
(Ukraine, Kiev)
E-mail: `ruslcomp@mail.ru`

Given a permutational wreath product sequence of cyclic groups [1, 3] of order 2 we research a commutator width of such groups and some properties of its commutator subgroup. Commutator width of Sylow 2-subgroups of alternating group A_{2^k}, permutation group S_{2^k} and $C_p \wr B$ were founded. The result of research was extended on subgroups $(Syl_2 A_{2^k})'$, $p > 2$. The paper presents a construction of commutator subgroup of Sylow 2-subgroups of symmetric and alternating groups. Also minimal generic sets of Sylow 2-subgroups of A_{2^k} were founded. Elements presentation of $(Syl_2 A_{2^k})'$, $(Syl_2 S_{2^k})'$ was investigated. We prove that the commutator width [2] of an arbitrary element of a discrete wreath product of cyclic groups C_{p_i}, $p_i \in \mathbb{N}$ is 1.

Lemma 1. *For any group B and integer $p \geq 2$, $p \in \mathbb{N}$ if $w \in (B \wr C_p)'$ then w can be represented as the following wreath recursion*

$$w = (r_1, r_2, \ldots, r_{p-1}, r_1^{-1} r_2^{-1} \ldots r_{p-1}^{-1} \prod_{j=1}^{k} [f_j, g_j]),$$

where $r_1, \ldots, r_{p-1}, f_j, g_j \in B$, and $k \leq cw(B)$.

Lemma 2. *An element $(g_1, g_2)\sigma^i \in G'_k$ iff $g_1, g_2 \in G_{k-1}$ and $g_1 g_2 \in B'_{k-1}$.*

Lemma 3. *For any group B and integer $p \geq 2$ inequality*

$$cw(B \wr C_p) \leq \max(1, cw(B))$$

holds.

Corollary 4. *If $W = C_{p_k} \wr \ldots \wr C_{p_1}$ then for $k \geq 2$ $cw(W) = 1$.*

Corollary 5. *Commutator width $cw(Syl_p(S_{p^k})) = 1$ for prime p and $k > 1$ and commutator width $cw(Syl_p(A_{p^k})) = 1$ for prime $p > 2$ and $k > 1$.*

Theorem 6. *Elements of $Syl_2 S'_{2^k}$ have the following form $Syl_2 S'_{2^k} = \{[f, l] \mid f \in B_k, l \in G_k\} = \{[l, f] \mid f \in B_k, l \in G_k\}$.*

Theorem 7. *Commutator width of the group $Syl_2 A_{2^k}$ equal to 1 for $k \geq 2$.*

Proposition 8. *The subgroup $(syl_2 A_{2^k})'$ has a minimal generating set of $2k - 3$ generators.*

References

[1] *R. Skuratovskii,* Generators and relations for Sylows p-subgroup of group S_n.(in ukranian) Naukovi Visti KPI. 2013, N. 4 pp. 94 -105.

[2] *Alexey Muranov,* Finitely generated infinite simple groups of infinite commutator width. arXiv:math/0608688v4 [math.GR] 12 Sep 2009.

[3] *R. V. Skuratovskii,* Structure and minimal generating sets of Sylow 2-subgroups of alternating groups. Source: https://arxiv.org/abs/1702.05784v2