Fuzzy metrization of the spaces of idempotent measures

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Let X be a compact Hausdorff space. As usual, by $C(X)$ we denote the space of continuous functions on X endowed with the sup-norm. For any $c \in \mathbb{R}$, we denote by c_X the constant function on X taking the value c.

Let $\mathbb{R}_{\max} = \mathbb{R} \cup \{-\infty\}$. We consider the natural order on $\mathbb{R}_{\max}$. We also use the following traditional notation from idempotent mathematics: $\oplus$ for max and $\odot$ instead of $+$ (this may concern either numbers or functions).

A functional $\mu \colon C(X) \to \mathbb{R}$ is said to be an *idempotent measure* (Maslov measure) if the following holds: (1) $\mu(c_X) = c$; (2) $\mu(\varphi \oplus \psi) = \mu(\varphi) \oplus \mu(\psi)$; (3) $\mu(\lambda \odot \varphi) = (\lambda \odot \mu\varphi)$. Remark that the notion of idempotent measure is a counterpart of the notion of probability measure in the so called idempotent mathematics.

The set $I(X)$ is endowed with the weak* topology (see [1]). Actually, the construction I determines a functor in the category of compact Hausdorff spaces. The space $I(X)$ is known to be metrizable for metrizable X.

The aim of the talk is to provide a fuzzy metrization of the space $I(X)$ for fuzzy metrizable X. We use the notion of fuzzy metric in the sense of George and Veeramani [2]. Recall that a triple $(X, M, *)$ is said to be a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X \times X \to (0, \infty)$ satisfying the following conditions for all $x, y, z \in X$ and $s, t \in (0, \infty)$:

(1) $M(x, y, t) > 0$;
(2) $M(x, y, t) = 1$ if and only if $x = y$;
(3) $M(x, y, t) = M(y, x, t)$;
(4) $M(x, y, t) * M(y, z, s) = M(x, z, t + s)$;
(5) the function $M(x, y, \cdot) \colon (0, \infty) \to [0, 1]$ is continuous.

Our approach is based on the notion of density of idempotent measure. Also, we use the fuzzy Hausdorff metric on the hyperspaces of fuzzy metric spaces [3]. We also show that the obtained construction determines a functor in the category of compact fuzzy metric spaces and non-expanding maps. We establish some properties of this functor related to the notion of monad.

References

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