Deformation of functions on orientable surfaces by symplectic diffeomorphisms

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Let M be a compact orientable surface and ω be a volume from M. We will study the right action of the group $Symp(M, \omega)$ of symplectic difeomorphisms on the space $C^{\infty}(M, \mathbb{R})$ of smooth functions on M.

Let $f : M \to \mathbb{R}$ be a C^{∞} Morse function, H be the Hamiltonian vector field of f with respect to ω, and $\mathcal{Z}_{\omega}(f)$ be the set of all C^{∞}-functions $M \to \mathbb{R}$, taking constant values along orbits of H. Then $\mathcal{Z}_{\omega}(f)$ is an abelian group with respect to point wise addition.

Further, let $\mathcal{S}(f, \omega) = \{h \in Symp(M, \omega) \mid f \circ h = f\}$ be the stabilizer of f with respect to the right action of the group $Symp(M, \omega)$. Thus $\mathcal{S}(f, \omega)$ consists of diffeomorphism mutually preserving f and ω. Let also $\mathcal{S}_0(f, \omega)$ be the identity path component of $\mathcal{S}(f, \omega)$ with respect to C^{∞} topology.

We will prove that there exists a canonical epimorphism of topological groups:

$$\phi : \mathcal{Z}_{\omega}(f) \to \mathcal{S}_0(f, \omega),$$

which is an isomorphism whenever f has at least one saddle critical point, and an infinite cyclic covering otherwise.

In particular, $\mathcal{S}_0(f, \omega)$ is an abelian group, being either contractible of homotopy equivalent tothe circle.

References

[1] Sergiy Maksymenko, *Symplectomorphisms of surfaces preserving a smooth function, I.* Topology and its Applications, vol. 235, (2018) 275-289