On the Cauchy problem for matrix factorizations of the Helmholtz equation

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Using our previous results, we show an explicit construction of the Carleman matrix for matrix factorisations of the Helmholtz equation in an unbounded planar domain. This allows one to get a formula for a regularized solution of the Cauchy problem with data on the finite part of the boundary.

It is known that the Cauchy problem for elliptic equations is unstable relatively small change in the data, i.e., is incorrect (Hadamard's example). In unstable problems the image of the operator is not closed, therefore the solvability condition can not be written in terms of continuous linear functionals. Thus, in the Cauchy problem for elliptic equations with data on a part of the boundary of the region, the solution is usually unique, the problem is solvable for an everywhere dense set of data, but this set not closed. Consequently, the theory of solvability of such problems is essentially It is more difficult and deeper than the theory of solvability of the Fredholm equations [8].

In this paper we construct a family of vector-functions $U_{\sigma(\delta)}(x) = U(x, f_\delta)$ depending on a parameter σ and it is proved that, under certain conditions and a special choice of the parameter $\sigma = \sigma(\delta)$; as $\delta \to 0$, the family $U_{\sigma(\delta)}(x)$ converges in the usual sense to a solution $U(x)$ at the point $x \in G$.

Following A.N. Tikhonov [5], a family of vector-functions $U_{\sigma(\delta)}(x)$ is called a regularized solution of the problem. A regularized solution determines a stable method of approximate solution of the problem. For special domains, the problem of extending bounded analytic functions in the case when the data is specified exactly on a part of the boundary was considered by Carleman [1]. The researches of T. Carleman were continued by G.M. Goluzin and V.I. Krylov [4]. A multidimensional analogue of Carleman's formula for analytic functions of several variables was constructed in [3]. The use of the classical Green's formula for constructing a regularized solution of the Cauchy problem for the Laplace equation was proposed by Academician M.M. Lavrent'ev [2], in his famous monograph. Extending Lavrent'ev idea, Yarmukhamedov constructed the Carleman function for the Cauchy problem for the Laplace equation ([6]-[7]). The Cauchy problem for the multidimensional Lame system is considered by O.I. Makhmudov and I.E. Niyozov [9]. The construction of the Carleman matrix for elliptic systems was carried out by Sh. Yarmukhamedov, N.N. Tarkhanov, O.I. Makhmudov, I.E. Niyozov and others.

In many well-posed problems for a system of equations of elliptic type of the first order with constant coefficients, the factorizing operator of Helmholtz, the calculation of the value of the vector function on the whole boundary is inaccessible. Therefore, the problem of reconstructing, solving a system of equations of elliptic type of the first order with constant coefficients, the factorizing operator of Helmholtz, is one of the topical problems in the theory of differential equations ([10]-[12]).

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