In a natural topological sense a typical linear nonhomogeneous differential equation in the ring $Z[[x]]$ has no solutions from $Z[[x]]$.

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Let $Z[[x]]$ be a ring of formal power series with integer coefficients. On $Z[[x]]$ we consider the topology of coefficientwise convergence (see [1], Ch.1, section 0.4). Let $b \in Z$, $b \neq 0$ and $f \in Z[[x]]$. The following implicit linear nonhomogeneous differential equation

$$by' + f(x) = y \tag{1}$$

is considered.

If $f(x)$ is a polynomial with integer coefficients, then the equation (1) has a unique solution as a polynomial with integer coefficients. If $f \in Z[[x]]$ is a nontrivial formal power series, then this equation can has no solutions in the ring $Z[[x]]$. For example, the equation $y' + 1 + x + x^2 + \ldots = y$ has no a solution as a formal power series with integer coefficients.

We denote by M the set of all formal power series $f \in Z[[x]]$ for which the equation (1) has a solution in the ring $Z[[x]]$.

The next main result is obtained

Theorem 1. *M is an uncountable dense submodule in the ring $Z[[x]]$. Moreover, $Z[[x]] \setminus M$, i.e. the set of those elements in the ring $Z[[x]]$ for which the equation (1) has no solutions from $Z[[x]]$, is a dense set of the class G_δ.*

References

[1] H. Grauert, R. Remmert. Analytische Stellenalgebre. *Springer-Verlag Berlin Heidelberg New York*, 1971.