b-bimorphisms

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Let X be an Archimedean vector lattice. $X^{\sim}$ denotes the order dual of X and $X^{\sim\sim}$ denotes the order bidual of X. By $(X^{\sim})_n^{\sim}$ we denote the order continuous bidual of X . The canonical mapping σ of X into $X^{\sim\sim}$ is defined by $\sigma(x)(f) = f(x) = x^{\sim}(f)$ for all $f \in X^{\sim}$.Here, $x^{\sim}$ defines an order continuous algebraic lattice homomorphism on $X^{\sim}$ and canonical image $\sigma(X)$ of X is a subalgebra of $(X^{\sim})_n^{\sim}$.The band generated by $\sigma(X)$ is order dense in the order continuous bidual $(X^{\sim})_n^{\sim}$ of X.

Definition 1. Let X be an Archimedean vector lattice. A bilinear mapping $T : X \times X \to X$ is called a b-bimorphism if $x \wedge y = 0$ and $x \wedge z = 0$ in X imply $x \wedge T(y, z) = 0$.

Every biorthomorphism is a b-bimorphism by the definition.

Theorem 2. *Let X be an Archimedean vector lattice. If $T : X \times X \to X$ is a b-bimorphism, then the triadjoint of T, $T''' : (X^{\sim})_n^{\sim} \times (X^{\sim})_n^{\sim} \to (X^{\sim})_n^{\sim}$ is a b-bimorphism.*

As a result of this study, we obtain that if A is a b-algebra, then the order continuous bidual of A is a b-algebra. Also, as a special case, the following result is presented, [7]

Corollary 3. *If a b-algebra A has positive squares , then the order bidual of A is a b-algebra.*

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