On the existence of a global diffeomorphism between Fréchet spaces

Kaveh Eftekharinasab
(Institute of mathematics of NAS of Ukraine)
E-mail: `kaveheft@gmail.com`

We provide sufficient conditions for the existence of a global diffeomorphism between tame Fréchet spaces. We prove a version of Mountain Pass theorem which plays a key ingredient in the proof of the main theorem. We apply differentiability in the sense of Micheal and Bastiani.

Theorem 1 (The Mountain Pass Theorem)**.** *Let F be a Fréchet space and $\phi : F :\to \mathbb{R}$ a C^1- functional satisfying the Palais-Smale condition. Assume $f \in F$ satisfies $\inf_{n\in\mathbb{N}} \parallel f \parallel^n > r > 0$ for a real number r and the geometric condition $\inf_{p\in S(0,r)} I(p) > \max\{\gamma(0), \gamma(f)\} = a$, where $S(0,r)$ is a sphere of the radius r. Then ϕ has a critical value $c \geqq a$ which can be characterized as*

$$c = \inf_{\gamma\in\Gamma} \max_{t\in[0,1]} \phi(\gamma(t)).$$

Where $\Gamma = \{\gamma \in C([0,1]; F) : \gamma(0) = 0, \gamma(1) = f \in F\}$.

Let E and F be tame Fréchet spaces and $\tau : E \to F$ a smooth tame map. Assume that a c^1-functional $\iota(x)$ is such that it and and its derivative $\iota'(y)$ are zero if and only if x and y are zero. In addition, we suppose that the derivative $\tau'(e)f = k$ has a unique solution $f = \nu(e)k$ for all $e \in E$ and all k, and the family of inverses $\nu : E \times F \to E$ is a smooth tame map.

Theorem 2. *Assume that a smooth tame map $\tau : E \to F$ and a C^1-functional ι on F are as above. If the following conditions hold*

C1: *for any $f \in F$ the functional $\jmath : E \to \mathbb{R}$ given by*

$$\jmath(e) = \iota(\tau(e) - f)$$

satisfies the Palais-Smale condition;

C2: *there exist positive real numbers a, b and c such that for f in the disk $D(0, a)$*

$$\iota(f) \geqq c\alpha^b \text{ where } \alpha = \sup_{n\in\mathbb{N}} \parallel f \parallel^n . \tag{1}$$

Then τ is a diffeomorphism.