Order continuity properties of lattice ordered algebras

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First, we give some fundamental notions.

Definition 1. A linear ordering of a real linear space X is an ordering satisfying these conditions:

(1) $x \leq y$ implies $x + z \leq y + z$, for all x, y, z in X,
(2) $x \leq y, \lambda \geq 0$ implies $\lambda x \leq \lambda y$.

Definition 2. An ordered linear space is a linear space with a linear ordering. Let A be an algebra with the unit e and A^+ be positive cone of A. For elements x, y of A $x \leq y$ means $x - y \in A^+$. A is an ordered linear space with this ordering.

Definition 3. If $xy \geq 0$ whenever $x \geq 0, y \geq 0$, then A is called an ordered algebra. If A is a Banach algebra with a closed cone A^+, then A is called an ordered Banach algebra.

Definition 4. If A is a real vector lattice and is associative but not necessarily commutative or unital algebra such that the multiplication and the partial ordering in A are compatible, i.e. $x, y \in A^+ \Rightarrow xy \in A^+$, then A is called a lattice-ordered algebra($l-$algebra).

Definition 5. An $l-$algebra is called

(1) a $d-$algebra whenever the multiplications by positive elements are lattice (Riesz) homomorphisms of A, that is, $(x \vee y)z = xz \vee yz$ and $z(x \vee y) = zx \vee zy$ for all $x, y \in A, z \in A^+$.
(2) an almost $f-$algebra if $x \wedge y = 0$ implies $xy = 0$.
(3) an $f-$algebra if $x \wedge y = 0$ implies $xz \wedge y = zx \wedge y = 0$ for all $z \in A^+$.

In this work, we mainly deal with lattice ordered algebras such as $f-$algebras, $d-$algebras and almost $f-$algebras and their properties.

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