Semitopological graph inverse semigroups

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We investigate locally compact semitopological graph inverse semigroups and obtain the following result:

Theorem 1. *Let E be a strongly connected directed graph which contains a finite amount of vertices. Then a Hausdorff locally compact semitopological graph inverse semigroup $G(E)$ over graph E is either compact or discrete.*

The above result generalizes results of Gutik [2] and Bardyla [1] who proved the above dichotomy for locally compact semitopological polycyclic monoids $\mathcal{P}_1$ and $\mathcal{P}_\lambda$, respectively.

The following theorem characterizes graph inverse semigroup which admit compact Hausdorff semigroup topology.

Theorem 2. *Let $G(E)$ be a compact semitopological semigroup. Then the following conditions are equivalent:*

(1) $G(E)$ is a topological inverse semigroup;
(1) the set $I_e = \{u \in \mathrm{Path}(E) \mid r(u) = e\}$ is finite for each vertex e;
(1) each $\mathcal{D}$-class is finite in $G(E)$;
(1) $G(E)$ does not contain isomorphic copies of the bicyclic monoid and an infinite semigroup of $X\times X$-matrix units.

Also we construct (in canonical way) the coarsest Hausdorff inverse semigroup topology $\tau_{\min}$ on each graph inverse semigroup $G(E)$. Moreover, the following theorem holds:

Theorem 3. *For each directed graph E topological semigroup $(G(E), \tau_{\min})$ embeds into the polycyclic monoid $(\mathcal{P}_{|G(E)|}, \tau_{\min})$.*

References

[1] Serhii Bardyla. Classifying locally compact semitopological polycyclic monoids. *Math. Bulletin of the Shevchenko Scientific Society*, 13 : 21–28, 2016.
[2] Oleh Gutik. On the dichotomy of the locally compact semitopological bicyclic monoid with adjoined zero. *Visn. L'viv. Univ., Ser. Mekh.-Mat.*, 80 : 33–41, 2015.