On the problem of product of inner radii symmetric non-overlapping domains

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Let $\mathbb{N}$, $\mathbb{R}$ be the sets of natural and real numbers, respectively, $\mathbb{C}$ be the complex plane, $\overline{\mathbb{C}} = \mathbb{C}\bigcup\{\infty\}$ be its one-point compactification. Let $r(D,a)$ be the inner radius of the domain $D \subset \overline{\mathbb{C}}$ with respect to a point $a \in D$. The inner radius is a generalization of the conformal radius for multiply connected domains. The inner radius of the domain D is associated with the generalized Green function $g_D(z,a)$ of the domain D by the relations

$$g_D(z,a) = \ln\frac{1}{|z-a|} + \ln r(D,a) + o(1), \quad z \to a,$$

$$g_D(z,\infty) = \ln|z| + \ln r(D,\infty) + o(1), \quad z \to \infty.$$

The system of non-overlapping domains is called a finite set of arbitrary domains $\{D_k\}_{k=0}^n$, $n \in \mathbb{N}$, $n \geqslant 2$ such that $D_k \subset \overline{\mathbb{C}}$, $D_k \cap D_m = \emptyset$, $k \neq m$, $k,m = \overline{0,n}$.

Denote by $\alpha_k := \dfrac{1}{\pi}\arg\dfrac{a_{k+1}}{a_k}$, $\alpha_{n+1} := \alpha_1$, $k = \overline{1,n}$, $\sum\limits_{k=1}^{n}\alpha_k = 2$.

Theorem 1. *For any $\gamma > 1$ there exists $n_0(\gamma) \in \mathbb{N}$, such that for any $n \geqslant n_0(\gamma)$, and system of points $A_n = \{a_k\}_{k=1}^n$, $|a_k| = 1$, and system of pairwise non-overlapping domains $\{D_k\}_{k=0}^n$, $0 \in D_0$, $a_k \in D_k \subset \overline{\mathbb{C}}$, such that domains $\{D_k\}_{k=1}^n$ are symmetric with respect to a unit circle, the following inequality holds*

$$r^\gamma(D_0,0)\prod_{k=1}^{n} r(D_k,a_k) \leqslant r^\gamma\left(D_0^{(0)},0\right)\prod_{k=1}^{n} r\left(D_k^{(0)},a_k^{(0)}\right).$$

The equality is attained if $a_k = a_k^{(0)}$ and $D_k = D_k^{(0)}$, $k = \overline{0,n}$, $a_0 = 0$, where $a_k^{(0)}$ and $D_k^{(0)}$ are, respectively, poles and circular domains of the quadratic differential

$$Q(w)dw^2 = -\frac{\gamma w^{2n} + 2(n^2-\gamma)w^n + \gamma}{w^2(w^n-1)^2}\,dw^2.$$

Rerefences

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