About some properties of functions determined as transformations from W^n to W^m-representation

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In this short article we describe the main properties of W^n-representation of points of unit hypercube. Our main goal is to show main properties of functions determined as transformations from W^n-representation of unit square to W^m-representation of points from unit interval [1].

1. Algorithm for construction of W^n-representation

At the beginning we have the unit n-dimensional hypercube $I^n = [0,1]^n$. W^n-representation of I^n can be received using next steps.

1) Let us divide the I^n into r parts which are closed in R^n and their interiors don't intersect. Lebesgue measures of new sets $\Delta_0, \Delta_1, ..., \Delta_{r-1}$ are $q_0, q_1, ..., q_{r-1}$ respectively.

2) Each set Δ_i is divided analogically into parts $\Delta_{i0}, ..., \Delta_{i[r-1]}$. The proportion of participles' Lebesgue measures remains invariant. After the second stage process continues and each set $\Delta_{\alpha_1...\alpha_k}$ is divided using the same rule.

If $k \to \infty$ then the Lebesgue measure of $\Delta_{\alpha_1...\alpha_k}$ must converge to zero.

2. Functions of transformation

Let us to define function of transformation W^n to W^m-representation.

Definition 1. Function of transformation from W^n to W^m-representation is a functional mapping which gives for each $x = \Delta^{W^n}_{\alpha_1...\alpha_k...}$ a unique $y = \Delta^{W^m}_{y(\alpha_1)...y(\alpha_k)...}$.

Remark 2. For uniqueness of such mapping we will consider to choose only the representation of $x \in E^n$ in which vector $(\alpha_1, ..., \alpha_k, ...)$ is minimal.

Definition 3. Function of transformation from W^n- to Q-representation is a functional mapping which gives for each $x = \Delta^{W^n}_{\alpha_1...\alpha_k...}$ a unique $y = \Delta^{Q}_{y(\alpha_1)...y(\alpha_k)...}$.

We received new results for the graphs of functions of transformation from W^n- to Q-representation.

Definition 4. Simple n-cube W^n-representation of I^n is W^n-representation which consists only of n-cubes with equal Lebesgue n-dimensional measure on each step (we enumerate particles from left higher corner and preserve orientation of numeration on next steps).

Theorem 5. *The graph of function of transformation from* simple n-cube W^n*−representation to s-adic number representation* [2] *is a fractal set with dimension* $log(4)/log(4^n) = 1/n$

Theorem 6. *The graph of function of transformation from* simple quadrate W^n*-representation to s-adic number representation is nowhere connected set.*

Definition 7. Connected n-cube W^n-representations of I^n is a class of W^n-representations such that:

1) W^n-representation consists only of figures with equal Lebesgue n-dimensional measure on each step (we enumerate particles from left higher corner and preserve orientation of numeration on next steps).

2) At least one point of I^n has continuum quantity of W^n-representations in this system.

Theorem 8. *The graph of function of transformation from* connected n-cube W^n*−representation to s-adic number representation is nowhere connected set.*

Our next goal is to receive new information about properties of functions of transformation from W^n- to W^m-representation and to formulate general theorems for their classification.

Rerefences

[1] Voloshyna V. *Properties and applications of W^n-representation of points from unit hypercube.* (in addition, TIMS, 2016)

[2] Pratsiovytyi M. V. *Fractal approach to researching of singular distributions.* National pedagogical university named after M. P. Dragomanov, Kyiv, 1998, 296 p.