More about set-theoretical entropies in generalized shifts

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For arbitrary map $f : A \to A$ and finite subset F of A [2]:

$$\mathrm{ent}_{set}(f,F) := \lim_{n\to\infty} \frac{\sharp(F \cup f(F) \cup \cdots \cup f^{n-1}(F))}{n}$$

exists and we call $\mathrm{ent}_{set}(f) := \sup\{\mathrm{ent}_{set}(f,F) : D \text{ is a finite subset of } A\}$, the set-theoretical entropy of $f : A \to A$.

Moreover, if $f : A \to A$ is surjective and finite fiber (i.e., $f^{-1}(x)$ is finite for all $x \in A$), then for finite subset F of A [5]:

$$\mathrm{ent}_{cset}(f,F) := \lim_{n\to\infty} \frac{\sharp(F \cup f^{-1}(F) \cup \cdots \cup f^{-(n-1)}(F))}{n}$$

exists and we call $\mathrm{ent}_{cset}(f) := \sup\{\mathrm{ent}_{cset}(f,F) : D \text{ is a finite subset of } A\}$, the contravariant set-theoretical entropy of $f : A \to A$.

Let's recall two well-known maps, one-sided shift $\underset{(x_n)_{n\in\mathbb{N}} \mapsto (x_{n+1})_{n\in\mathbb{N}}}{\{1,\dots,k\}^{\mathbb{N}} \to \{1,\dots,k\}^{\mathbb{N}}}$ and two-sided shift $\underset{(x_n)_{n\in\mathbb{Z}} \mapsto (x_{n+1})_{n\in\mathbb{Z}}}{\{1,\dots,k\}^{\mathbb{Z}} \to \{1,\dots,k\}^{\mathbb{Z}}}$. So for nonempty set Γ, arbitrary set X with at least two elements and self-map $\varphi : \Gamma \to \Gamma$ one may consider generalized shift $\sigma_\varphi : \underset{(x_\alpha)_{\alpha\in\Gamma} \mapsto (x_{\varphi(\alpha)})_{\alpha\in\Gamma}}{X^\Gamma \to X^\Gamma}$. It's evident that if X has topological (resp. group, linear vector space) structure, then X^Γ has topological (resp. group, linear vector space) structure too.

Suppose X is a finite discrete space with at least two elements, Γ is an infinite set and $\varphi : \Gamma \to \Gamma$ is an arbitrary map also equip X^Γ with product (pointwise convergence) topology, then $\sigma_\varphi : X^\Gamma \to X^\Gamma$ is topological chaotic (positive topological entropy) if and only if $\varphi : \Gamma \to \Gamma$ has positive set-theoretical entropy [2].

Moreover if X is a non-trivial finite abelian group and $\varphi : \Gamma \to \Gamma$ is finite fiber, then algebraic entropy of $\sigma_\varphi \restriction_{\bigoplus_{\alpha\in\Gamma} X} : \bigoplus_{\alpha\in\Gamma} X \to \bigoplus_{\alpha\in\Gamma} X$ interacts with contravariant set-theoretical entropy of $\varphi : \Gamma \to \Gamma$ [1, 3].

In this talk we take a short tour on some obtained results on generalized shifts including: Devaney chaotic, Li-Yorke chaotic, e-chaotic, P-chaotic,.... generalized shifts including the list of published papers on the subject. Our final attention will be on generalized shifts set-theoretical entropies [4], with more details and results.

Rerefences

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