Theory of gravity in the affine frame

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Let $e_a = h_a^\mu \partial_\mu$ by the affine frame in the Riemann space, $g_{ab} = e_a \cdot e_b$ and $F^a_{\mu\nu} = \partial_\nu h^a_\mu - \partial_\mu h^a_\nu$ – non-holonomity coefficients. Suppose, that γ^a_{bc} is coefficients of the torsion-free and metric-compatible affine connection in the affine frame e_a, so $F^a_{bc} = \gamma^a_{bc} - \gamma^a_{cb}$ and $g_{ab,c} = \gamma^{\cdot}_{acb} + \gamma^{\cdot}_{bca}$. The Riemann curvature scalar $R = \delta^{\mu\nu}_{ab} g^{bd}(\partial_\mu \gamma^a_{\nu d} + \gamma^a_{\mu c}\gamma^c_{\nu d})$, where $\delta^{\mu\nu}_{ab}$ - alternator, decompose into $R = -L_\gamma + \frac{1}{e}\partial_\mu(eV^\mu)$, where $e^2 = det\{g_{ab}h^a_\mu h^b_\nu\}$ and $V^\mu = \gamma^\mu_{cc} - \gamma^{c\mu}_{c\cdot}$. Function

$$L_\gamma = \delta^{\mu\nu}_{ab} g^{bd} \gamma^a_{\mu c} \gamma^c_{\nu d}$$

plays the role of Lagrangian in the theory of gravity in the affine frame (TGAF).

Let's consider T^g-transformations:

$$\delta x^\mu = h^\mu_a t^a,$$
$$\delta h^b_\mu = -F^b_{\mu a} t^a - \partial_\mu t^a,$$
$$\delta g_{bc} = -g_{bc,a} t^a$$

with infinitesimal parameters t^a. Lagrangian L_γ is invariant under this transformations, so take place the strong Noether's identity

$$t^\mu_a + \nabla_\sigma B^{\mu\sigma}_a = -G^\mu_a,$$

where

$$t^\mu_a = B^{\sigma\mu}_b F^b_{\sigma a} + D^{bc\mu} \gamma^{\cdot}_{bac} - L_\gamma h^\mu_a$$

is the energy-momentum tensor of the gravitational field in TGAR,

$$B^{\mu\sigma}_a = \delta^{\mu\sigma}_{\rho\nu}(\gamma^{\rho\nu}_{a\cdot} + h^\rho_a V^\nu),$$

$$D^{bc\mu} = -(\gamma^{\mu bc} + \gamma^{\mu cb}) + h^\mu_d(g^{db}\gamma^{ac}_a + g^{dc}\gamma^{ab}_a) + g^{bc}V^\mu$$

and G^μ_a - Einstein tensor. On the gravitational extremal $G^\mu_a = \tau^\mu_a$, where τ^μ_a - energy-momentum tensor of matter fields, we obtain the equation for gravitation field in TGAR:

$$\partial_\sigma(eB^{\mu\sigma}_a) = -eT^\mu_a,$$

where $T^\mu_a = t^\mu_a + \tau^\mu_a$ is the complete energy-momentum tensor of the gravitational and matter fields, and $eB^{\mu\sigma}_a$ plays the role of its superpotential. This equation has the form of Maxwell equations and equations of gauge theory of gravity in the orthonormal frame [1].

Rerefences

[1] S. E. Samokhvalov, V. S. Vanyashin. Group theory approach to unification of gravity with internal symmetry gauge interactions. *Class. Quantum Grav.*, 8 : 2277–2282, 1991.