On nuclear operators with $trace\, V = 1$ and $V^2 = 0$

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In 1955, A. Grothendieck [2] has introduced the notion of the approximation property and has shown that *there exists a Banach space without the approximation property iff there exists an operator* $U : c_0 \to c_0$ *such that the (nuclear) trace of* U *equals 1 but* $U^2 = 0$ *identically* [2, Chap. I, "Proposition" 37. $(a') \Leftrightarrow (f'')$, pp. 170-171].

Recall that a Banach space X has the approximation property if the identity map id_X can be approximated, in the topology of compact convergence, by finite rank operators.

An operator $T : X \to X$ is said to be s-nuclear ($0 < s \leq 1$) if there are linear continuous functionals $(f_k) \subset X^*$ and elements $(x_k) \subset X$ so that $\sum ||f_k||^s\, ||x_k||^s < \infty$ and $T(x) = \sum f_k(x) x_k$ for every $x \in X$. They say "nuclear operator" if $p = 1$.

After Per Enflo's construction of a Banach space without the approximation property [1], it was shown (see e.g. [3, 10.4.5]) that there are not only nuclear operators in c_0 with the above property but also there exists a nuclear operator T in l^1 which is s-nuclear for every $s \in (2/3, 1]$ and such that $trace\, T = 1$ and $T^2 = 0$.

We discuss a way to get the nuclear operators of such a kind in the spaces l^p, for all $p > 1, p \neq 2$ (in c_0 for $p = \infty$):

Theorem 1. *Let* $p \in [1, \infty], p \neq 2$, $1/r = 1 + |1/2 - 1/p|$. *There exists a nuclear operator* V *in* l^p *(in* c_0 *for* $p = \infty$*) such that*

(1) V *is* s*-nuclear for each* $s \in (r, 1]$;
(2) V *is not* r*-nuclear;*
(3) trace $V = 1$ *and* $V^2 = 0$.

Theorem 2. *Theorem 1 is optimal with respect to* p *and* r.

Note that for $p = \infty$ we have $r = 2/3$.

We give also some applications.

Rerefences

[1] Per Enflo. A counterexample to the approximation property in Banach spaces. *Acta Mathematica*, 130 : 309–317, 1973.

[2] Alexander Grothendieck. *Produits tensoriels topologiques et éspaces nucléaires*, volume 16 of *Memoires of American Mathematical Society*. 1955.

[3] A. Pietsch. *Operator Ideals*. North Holland, 1980.