Field equations from geometric Killing spinors

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Spinors, with different and more general properties from vectors or tensors, represent fermionic fields or particles on their own. Because one can square them to construct differential forms they also have the power to represent bosonic objects in contrary to tensor fields. Spacetime spinor fields are characterized by the differential equations that they satisfy which are based on the spinor connection that is used for illustrating the dynamics of fermionic fields and the motion of fermionic particles in these fields apriori. Killing fermions are defined as the spinors satisfying the geometric Killing spinor equation

$$\nabla_X \psi = \lambda \widetilde{X}.\psi$$

and from our point of view are thought as physical more than geometrical. In a recent work [1] we worked out some properties of bilinears generated by twistors and Killing spinors. The Killing spinor case, accompanied by a data set, was more sophisticated and rich. All possible outcomes obtainable from the Killing spinor bilinears were determined by the restrictive reality conditions imposed on them for physical reasons. We then uncovered the *primitive set* of generating equations and they were seen to be

$$\nabla_{X_a}(\psi\overline{\psi})_p = 2\lambda\, e_a \wedge (\psi\overline{\psi})_{p-1}$$
$$\nabla_{X_a}(\psi\overline{\psi})_{p_*} = 2\lambda\, i_{X_a}(\psi\overline{\psi})_{p_*+1},$$

giving rise to the *principal set*

$$d(\psi\overline{\psi})_p = 0, \qquad d^\dagger(\psi\overline{\psi})_p = -2\lambda(n-p+1)(\psi\overline{\psi})_{p-1};$$
$$d(\psi\overline{\psi})_{p_*} = 2\lambda(p_*+1)(\psi\overline{\psi})_{p_*+1}, \qquad d^\dagger(\psi\overline{\psi})_{p_*} = 0.$$

Here p_* means that it has a different parity than p, i.e. $p_* + p$ is always odd and $0 \leq p, p_* \leq n$, n is the dimension of spacetime. In this talk [2] we will show that these equations can be used to generate interestingly the Klein-Gordon, Maxwell-like, Proca, Duffin-Kemmer-Petiau, Kähler and Rarita-Schwinger equations in curved spacetimes in a systematic manner. The Rarita-Schwinger case is based on the tool of tensor spinors over spacetime, that are higher dimensional half-spin representations of the Clifford group. When obtaining the correspondent field equations we also obtained a constraint that has to be satisfied which is directly related to the $3-\psi$ rule that appears in supersymmetric theories.

Rerefences

[1] Özgür Açık and Ümit Ertem, *Higher degree Dirac currents of twistor and Killing spinors in supergravity theories*, Class. Quantum Grav. **32**, 175007, 2015.
[2] Özgür Açık, *Field equations from Killing spinors*, Ankara University preprint, 2017.