The local density and the local weak density of N_τ^φ-kernel of a topological space X and superextensions

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Definition 1. *The weak density of a topological space X is the smallest cardinal number $\tau \geq \aleph_0$ such that there is a π-base in X coinciding with τ centered systems of open sets, i.e. there is a π-base $B = \bigcup\{B_\alpha : \alpha \in A\}$, where B_α is a centered system of open sets for each $\alpha \in A$ and $|A| = \tau$ [1].*

The weak density of a topological space X is denoted by $wd(X)$. If $wd(X) = \aleph_0$ then we say that a topological space X is weakly separable.

Definition 2. *We say that a topological space X is locally separable at a point $x \in X$ if x has a separable neighborhood [2].*

A topological space is locally separable if it is locally separable at each point $x \in X$.

Definition 3. *We say that a topological space X is locally τ-dense at a point $x \in X$ if τ is the smallest cardinal number such that x has a τ -dense neighborhood in X.*

The local density at a point x is denoted by $ld(x)$. The local density of a space X is defined as the supremum of all numbers $ld(x)$ for $x \in X$; this cardinal number is denoted by $ld(X)$.

Definition 4. *A topological space is locally weakly τ dense at a point $x \in X$ if τ is the smallest cardinal number such that x has a neighborhood of weak density τ in X*

The local weak density at a point x is denoted by $lwd(x)$.

The local weak density of a topological space X is defined with following way: $lwd(X) = \sup\{lwd(x) : x \in X\}$.

A system $\xi = \{F_\alpha : \alpha \in A\}$ of closed subsets of a space X is called *linked* if any two elements from ξ intersect. Any linked system can be complemented to a maximal linked system (MLS), but this complement is, as a rule, not unique [3].

Proposition 1 [3]. *A linked system ξ of a space X is a MLS iff it possesses the following completeness property:*

if a closed set $A \subset X$ intersects with any element form ξ, then $A \in \xi$.

Denote by λX the set of all MLS of the space X. For an open set $U \subset X$, set

$$O(U) = \{\xi \in \lambda X : \text{ there is an } F \in \xi \text{ such that } F \subset U\}.$$

The family of subsets in the form of $O(U)$ covers the set λX $(O(X) = \lambda X)$, that's why it forms an open subbase of the topology on λX. The set λX equipped with this topology is called *the superextension* of X.

A.V. Ivanov defined the space NX of complete linked systems (CLS) of a space X in a following way:

Definition 5. A linked system M of closed subsets of a compact X is called *a complete linked system* (a CLS) if for any closed set of X, the condition

"Any neighborhood OF of the set F consists of a set $\Phi \in M$"

implies $F \in M$[2].

A set NX of all complete linked systems of a compact X is called *the space NX of CLS of X*. This space is equipped with the topology, the open basis of which is formed by sets in the form of $E = O(U_1, U_2, \dots, U_n)\langle V_1, V_2, \dots, V_s\rangle = \{M \in NX :$ for any $i = 1, 2, \dots, n$ there exists $F_i \in M$ such that $F_i \subset U_i$, and for any $j = 1, 2, \dots, s$, $F \cap V_j \neq \varnothing$ for any $F \in M\}$, where $U_1, U_2, \dots, U_n$, $V_1, V_2, \dots, V_s$ are nonempty open in X sets [4].

Definition 6. Let X be a compact space, φ be a cardinal function and τ be an arbitrary cardinal number. We call *an N_τ^φ - kernel* of a topological space X the space

$$N_\tau^\varphi X = \{M \in NX : \exists F \in M : \varphi(F) \leq \tau\}.$$

Theorem 7. *Let X be an infinite T_1-space and $h\varphi(X) \leq \tau$. Then*
1) $ld(\lambda X) = ld(N_\tau^\varphi X)$;
2) $lwd(\lambda X) = lwd(N_\tau^\varphi X)$.

Rerefences

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[4] Ivanov A. V. *A space of complete linked systems*, volume 27 of *Siberian Mathematical Journal*. 1986. pp. 863-875.