Polyadic topology on Z and linear differential equations in the ring $Z[[x]]$

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Let $Z[[x]]$ be a ring of formal power series with integer coefficients. On Z we consider the polyadic topology (see [1], Ch.III, section 3.5 and [2]) and on $Z[[x]]$ we consider the topology of coefficientwise convergence (see [3], Ch.1, section 0.4).

Let $b \in Z$ and $f(x) \in Z[[x]]$. A question on solutions of the following implicit linear nonhomogeneous differential equation $by' + f(x) = y$ in the ring $Z[[x]]$ is studied. The next main results are obtained.

1. The equation $y' + 1 + x + x^2 + \ldots = y$ has no a solution as a power series with integer coefficients.
2. By the concept of the polyadic sum of integers (see [1], Ch.III, section 3.5), a necessary and sufficient condition for the existence of a solution of the differential equation $by'(x) + f(x) = y(x)$ as a power series with integer coefficients was found.
3. If the equation $by'(x) + f(x) = y(x)$ has a solution $y(x)$ from $Z[[x]]$ then
$$y(x) = f(x) + bf'(x) + b^2 f''(x) + \ldots,$$
and this series converges in the topology of coefficient-wise convergence.

Rerefences

[1] A.G. Postnikov. Introduction to Analytic Number Theory (Translations of Mathematical Monographs). *American Mathematical Society*, 1988, 320 p.
[2] V.G. Chirsky. Arithmetic properties of integer polyadic numbers. *Chebyshevskii Sbornik*, 2015, 307 p.
[3] H. Grauert, R. Remmert. Analytische Stellenalgebre. *Springer-Verlag Berlin Heidelberg New York*, 1971.