# Some cardinal and topological properties of $N_\tau^\varphi$-kernel of a topological space $X$ and superextensions

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A system $\xi = \{F_\alpha : \alpha \in A\}$ of closed subsets of a space $X$ is called *linked* if any two elements from $\xi$ intersect. Any linked system can be complemented to a maximal linked system (MLS), but this complement is, as a rule, not unique [1].

**Proposition 1.** [1]. *A linked system $\xi$ of a space $X$ is a MLS iff it possesses the following completeness property: if a closed set $A \subset X$ intersects with any element form $\xi$, then $A \in \xi$.*

Denote by $\lambda X$ the set of all MLS of the space $X$. For an open set $U \subset X$, set

$$O(U) = \{\xi \in \lambda X : \text{ there is an } F \in \xi \text{ such that } F \subset U\}.$$

The family of subsets in the form of $O(U)$ covers the set $\lambda X$ $(O(X) = \lambda X)$, that's why it forms an open subbase of the topology on $\lambda X$. The set $\lambda X$ equipped with this topology is called *the superextension* of $X$.

A.V. Ivanov defined the space $NX$ of complete linked systems (CLS) of a space $X$ in a following way:

**Definition 2.** A linked system $M$ of closed subsets of a compact $X$ is called *a complete linked system* (a CLS) if for any closed set of $X$, the condition

- any neighborhood $OF$ of the set $F$ consists of a set $\Phi \in M$

implies $F \in M$, [2].

A set $NX$ of all complete linked systems of a compact $X$ is called *the space $NX$ of CLS of $X$*. This space is equipped with the topology, the open basis of which is formed by sets in the form of

$$\begin{aligned} E &= O(U_1, U_2, \dots, U_n)\langle V_1, V_2, \dots, V_s\rangle \\ &= \{M \in NX : \text{for any } i = 1, 2, \dots, n \text{ there exists } F_i \in M \text{ such that } F_i \subset U_i, \\ &\qquad \text{and for any } j = 1, 2, \dots, s,\ F \cap V_j \neq \varnothing \text{ for any } F \in M\}, \end{aligned}$$

where $U_1, U_2, \dots, U_n$, $V_1, V_2, \dots, V_s$ are nonempty open in $X$ sets [2].

**Definition 3.** Let $X$ be a compact space, $\varphi$ be a cardinal function and $\tau$ be an arbitrary cardinal number. We call *an $N_\tau^\varphi$ - kernel* of a topological space $X$ the space

$$N_\tau^\varphi X = \{M \in NX : \exists F \in M : \varphi(F) \leq \tau\}.$$

**Theorem 4.** *Let $X$ be a normal topological space and $h\varphi(X) \leq \tau$. Then the following assertions are equivalent:*

*1) $t(\lambda(\lambda X)) \leq \aleph_0$;*
*2) $t(\lambda(X \times X)) \leq \aleph_0$ ;*
*3) $X$ is metrizable;*
*4) $\lambda X$ is metrizable;*

*5) $N^{\varphi}_{\tau}X$ is metrizable.*

## Rerefences

[1] Fedorchuk V. V., Filippov V. V. General Topology. Basic Constructions. *Fizmatlit, Moscow*, 2006.
[2] Ivanov A. V. *A space of complete linked systems*, volume 27 of *Siberian Mathematical Journal*. 1986. pp. 863-875.