

On a semitopological α -bicyclic semigroup

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We prove that α -bicyclic monoid \mathcal{B}_α is algebraically isomorphic to a semigroup of all order isomorphisms between the principal upper sets of the ordinal ω^α and prove that $\mathcal{B}_{\alpha+1}$ is isomorphic to the Brook extension of the semigroup \mathcal{B}_α . We prove that for every ordinal α for every $(a, b) \in \mathcal{B}_\alpha$ if either a or b is a non-limit ordinal then (a, b) is an isolated point in the semitopological \mathcal{B}_α . We show that for every ordinal $\alpha < \omega + 1$ every locally compact semigroup topology on \mathcal{B}_α is discrete. However, we construct an example of a non-discrete locally compact topology τ_{lc} on $\mathcal{B}_{\omega+1}$ such that $(\mathcal{B}_{\omega+1}, \tau_{lc})$ is a topological inverse semigroup. Also, for every positive integer n we describe all locally compact topologies on the semitopological \mathcal{B}_n . In particular we show that there exist exactly n distinct locally compact topologies on the semitopological n -bicyclic monoid \mathcal{B}_n .

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