On almost contact metric hypersurfaces in special Hermitian manifolds

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1. The well-known classification of the almost Hermitian structures on first order differential-geometrical invariants can be rightfully attributed to the most significant results obtained by the outstanding American mathematician Alfred Gray and his Spanish colleague Luis M. Hervella. According to this classification, all the almost Hermitian structures are divided into 16 classes. Analytical criteria for each concrete structure to belong to one or another class have been obtained [1].

The class of special Hermitian manifolds (or W_3-manifolds, using Gray-Hervella notation) has been studied not so detailed as other so-called “small” Gray-Hervella classes of almost Hermitian manifolds. Some dozens of significant works are devoted to the nearly Kählerian, almost Kählerian and locally conformal Kählerian manifolds, but much less of articles are written about special Hermitian manifolds.

We remark also that the present work is a continuation of researches of the authors in the area of Hermitian manifolds, mainly six-dimensional (see, for example, [2], [3], [4], [5], [6] and others).

2. As it is known, an almost Hermitian manifold is a $2n$-dimensional manifold M^{2n} with a Riemannian metric $g = \langle \cdot, \cdot \rangle$ and an almost complex structure J. Moreover, the following condition must hold

$$\langle JX,\ JY \rangle = \langle X,\ Y \rangle\,, \quad X, Y \in \aleph(M^{2n}),$$

where $\aleph(M^{2n})$ is the module of smooth vector fields on M^{2n} [7].

We recall that the fundamental (or Kählerian) form of an almost Hermitian manifold is determined by the relation

$$F(X,\ Y) = \langle X,\ JY \rangle\,, \quad X, Y \in \aleph(M^{2n}).$$

where $X, Y, Z \in \aleph(M^{2n})$. A special Hermitian structure in addition must satisfy the condition $\delta\, F = 0$, where δ is the codifferentiation operator [7].

3. The main results are the following:

1) The Cartan structural equations of the general type almost contact metric structure on an oriented hypersurface in a special Hermitian manifold are obtained;

2) The Cartan structural equations of some important kinds of almost contact metric structures (cosymplectic, Sasaki, Kenmotsu etc.) on an oriented hypersurface in a special Hermitian manifold are selected;

3) A characterization in terms of the type number of some important kinds of almost contact metric structures (cosymplectic, Sasaki, Kenmotsu etc.) on hypersurfaces in special Hermitian manifolds is obtained;

4) A criterion of the minimality of such hypersurfaces in the terms of their type number is established.

These results are detailed for six-dimensional planar submanifolds [4], [5] of Cayley algebra that carry special Hermitian structures.

Rerefences

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