Tuning role of surface slices within urbanism-index fuzzy system hypersurfaces

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1. INTRODUCTION AND PRELIMINARIES

For a long time, statistical and regression models were the most common analytical methods which, due to linearly dealing with the modelling phenomena, could not tract over-the-time changes with acceptable accuracy. However, recently, fuzzy and neural systems showed their ability to model nonlinear and complex phenomena, being successfully applied in various fields, including urbanism ([3, 1, 2]).

It is noteworthy that the geometry exhibited by the models based on fuzzy inference systems (FIS) proved to be an important tool for both model design and tuning, and for further prediction purposes. In such systems, the dependence of outputs on the input readings - regarded as parameters - naturally provide non-smooth Monge-type *hypersurfaces*.

In this work, by using FIS, we consider a model which uses least amount of metric information to predict the urbanism index of a city area. For such a system, we describe the factors which provide changes of the 2-parameter slices of the FIS hypersurface, viewed as Monge chart surfaces, and tools to measure the changes. We note that these slices are rich in information and might strongly differ, depending on various factors, e.g., used membership functions MF, Inference Rules, and defuzzification methods. As well, we analyze the relevance of the inferred variations of the the slice surfaces, and illustrate the theory by the metric-oriented urbanism index model, based on the recent statistics of the Bucharest city area.

We shall further consider a FIS of Mamdani type [5] (firstly introduced in [7]), which has the input linguistic variables $\mathcal{V} = \{v_1, \ldots, v_n\}$ $(n \ge 1)$ and one output variable w, having the corresponding input domains $\mathcal{D} = \{D_1, \ldots, D_n\} \subset \mathcal{P}(\mathbb{R})$ and respectively $D_* \subset \mathbb{R}$. For such FIS, both $\mathcal{V} \cup \{w\}$ with their MF, and the inference rules (IR), provide for each reading set $(t_1, \ldots, t_n) \in D_1 \times \ldots \times D_n$, after the defuzification of the aggregate output set (AS), the value $f(t_1, \ldots, t_n) \in D_*$. The image $\Sigma = f(D_1 \times \ldots \times D_n) \subset \mathbb{R}^{n+1}$ then represents an n-dimensional parametrized \mathcal{C}^0 -hypersurface of Monge type, which exhibits a prominent nonsmooth character. We further exemplify the role played by the geometry of Σ , by considering the surface slices $\Sigma_{ij} = \Sigma_{|t_k=c_k, k \in \overline{1,n} \setminus \{i,j\}}$, for $1 \le i < j \le n$.

2. FIS URBANISM-INDEX MODEL. SLICE SURFACE GEOMETRY VS. TUNING AND PROGNOSIS

To exemplify the tuning and prognostic role of slice-surfaces, we design a simplified model which mainly uses metric-type input linguistic variables built on the statistical official data of the Bucharest city area, inspired by a similar model (Boston, [2]). In this model, we fix n = 5, and the five input variables (whose domains are determined by the 1993-2015 time range data):

- (1) v_1 : City area (in hectares)¹; $D_1 = [15200, 24000];$
- (2) v_2 : City green area (in hectares); $D_2 = [4100, 4900];$
- (3) v_3 : City streets length (in kilometers)²; $D_3 = [1800, 3500]$;
- (4) v_4 : Modernized streets length (in kilometers); $D_4 = [800, 2600]$;
- (5) v_5 : City population (in units); $D_5 = [2100000, 2250000] \cap \mathbb{N};$
- (6) w: Urbanism index coefficient (UI) output linguistic variable (LV); $D_* = [0, 1]$.

¹Correspondence with non-SI units: 1 ha=2.471 acres.

²Correspondence with non-SI units: 1 km=0.621 miles=3281 feet.

The associated membership functions (which cover the attributes very low, low, medium, high, very high) have the support determined by the time-scale measurements within the corresponding range, and are of type trapezoid-triangles-trapezoid or Gaussian. The inference rules are estimated based on existing models from literature (e.g., [2, 3, 1, 6]), the considered FIS is of Mamdani type, and the defuzzification is of type COG and MOM.

By using Matlab programming and its Fuzzy Logic Toolbox facilities, one can observe and use the geometric relevance of the accurate numerically sampled slice-surfaces Σ_{ij} , for investigating the influence of a pair of the $i, j \, LV$ on the UI, under constant readings of the remaining 3 ones. This proves to be useful in such FIS-tuning situations, when both the pair (i, j) and the complementary readings for $k \in \overline{1, 5} \setminus \{i, j\}$ are fixed, as follows:

- the change of the supports of the MFs of the two linguistic variables when their impact on the UI is reconsidered, without changing the MF shape;
- the change of the shapes of the membership functions of the two linguistic variables when their shape and smoothness changes;
- the change of the inference rules which affect the two LV when their role in producing the UI is either modified as adjoint inputs/output, or changed as weight;
- the change of the defuzzification method when changing the dependence of UI in terms of the maximal range of the aggregate MF subset, vs. considering the whole aggregate MF;
- the extension of input domains \mathcal{D} in *predictions* when the two parameters predict a variation of the urbanism index (UI), whose variation can be experimentally estimated.

In each case, the newly obtained surface Σ'_{ij} allows the user to anticipate the change of the urbanism index. The change may be evaluated by considering the *distance* $d(\Sigma'_{ij}, \Sigma_{ij})$ between the two surfaces, which allows to indicate the type of tuning of the *FIS*, in order to finally produce a closer *UI* to its expected real estimate, and which ultimately empowers the *FIS* to become a valid tool for better customize the *UI* prognosis.

We illustrate the five situations in which the geometry of the area-population / UI slice surface changes due to the structural change of the two inputs, allowing a *proper* tuning of the *FIS*.

Our further research intends to develop and compare the FIS relative to other Romanian cities. The further goal aims to provide a tool FIS that would enable users to upload their input data, customize the rules and produce maps for a set of specific general objectives. We note that the integrated, multidisciplinary nature of this project maximizes opportunities to advance understanding of coupled natural and human systems in the urban context.

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