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Transformation Operators for Integrable Hierarchies with Additional Reductions

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New integrable reductions of the modified Kadomtsev–Petviashvili (mKP) hierarchy was obtained. We solve the so-called $\mathcal{D}$-Hermitian constrained mKP ($\mathcal{D}$HcmKP) hierarchy by using the dressing transformation technique. The dressing (transformation) operator for the $\mathcal{D}$HcmKP hierarchy is defined, and multicomponent derivative nonlinear Schrödinger equation was integrated as an example.

1 Introduction

We consider Lax–Zakharov–Schabat equations

$$\beta U_t - \alpha V_y + UV - VU = 0 \quad \Leftrightarrow \quad [\alpha\partial_y - U,\ \beta\partial_t - V] = 0,$$
$$\alpha, \beta \in \mathbb{C}, \quad \partial_y := \frac{\partial}{\partial y}, \quad \partial_t := \frac{\partial}{\partial t}, \tag{1}$$

in the algebra ζ of the microdifferential operators (MDO) [1].

$$U, V \in \zeta := \left\{ L = \sum_{i=-\infty}^{n(L)} a_i \mathcal{D}^i :\ a_i = a_i(x, y, t);\ i, n(L) \in \mathbb{Z} \right\}, \tag{2}$$

where MDO U, V satisfy additional constraints, which are concretely defined in the following section, and coefficients a_i are, in general, smooth $(N \times N)$ matrix-valued functions of $x, y, t, \in \mathbb{R}$. In the algebra MDO ζ (2) operation of multiplication is induced by the generalized Leibnitz rule

$$\mathcal{D}^n f := \sum_{j=0}^{\infty} \binom{n}{j} f^{(j)} \mathcal{D}^{n-j}, \quad n \in \mathbb{Z}, \qquad \mathcal{D}^m(f) := \frac{\partial^m f}{\partial x^m} = f^{(m)}, \quad m \in \mathbb{Z}_+, \tag{3}$$

where

$$\binom{n}{j} := \frac{n(n-1)\dots(n-j+1)}{j!}, \qquad \mathcal{D}^n \mathcal{D}^m := \mathcal{D}^{n+m}, \qquad n, m \in \mathbb{Z},$$

and f is the operator of multiplication by function $f(x, y, t)$, which belongs to the same functional space as the coefficients of microdifferential operators $L \in \zeta$ do. Lie's commutator in algebra ζ is defined as $[U, V] := UV - VU$, and Hermitian-conjugated operator $L^* := \sum_{i=-\infty}^{n(L)} (-1)^i \mathcal{D}^i a_i^*$, $a_i^* = \bar{a}^\top$, $(\alpha\partial_y)^* := -\bar{\alpha}\partial_y$, $(\beta\partial_t)^* := -\bar{\beta}\partial_t$.

2 Reduction of $\mathcal{D}$-Hermitian conjugation

Definition 1. We say that an operator $L \in \zeta$ is $\mathcal{D}$-Hermitian ($\mathcal{D}$-skew-Hermitian) if

$$L^* = \mathcal{D} L \mathcal{D}^{-1} \quad \left(L^* = -\mathcal{D} L \mathcal{D}^{-1}\right).$$

Definition 2. We say that an integral operator $W \in \zeta_{<1} := \left\{ L_{<1} := \sum_{i=-\infty}^{0} u_i \mathcal{D}^i \right\}$ is $\mathcal{D}$-unital if $W^{-1} = \mathcal{D}^{-1} W^* \mathcal{D}$.

Lemma 1. *Let $L^* = \mu \mathcal{D} L \mathcal{D}^{-1}$, $\mu = \pm 1$, and $W^{-1} = \mathcal{D}^{-1} W^* \mathcal{D}$. Then $\hat{L}^* = \mu \mathcal{D} \hat{L} \mathcal{D}^{-1}$, where $\hat{L} := W L W^{-1}$.*

Proof. $\hat{L}^* := \left(W L W^{-1}\right)^* = \left(W^{-1}\right)^* L^* W^* = \mu \mathcal{D} W \mathcal{D}^{-1} \mathcal{D} L \mathcal{D}^{-1} \mathcal{D} W^{-1} \mathcal{D}^{-1} = \mu \mathcal{D} \hat{L} \mathcal{D}^{-1}$. ■

Lemma 2. *Let h_i, g_i be smooth $(N \times K)$ matrix-valued functions of real variable $x \in \mathbb{R}$, $i = 1, 2$; $A = (a_{mn}) = \mathrm{const} \in \mathrm{Mat}_{K \times K}(\mathbb{C})$ and $a \in \mathbb{R} \cup \{\pm\infty\}$. Then*

$$h_1 \mathcal{D}^{-1} g_1^\top h_2 \mathcal{D}^{-1} g_2^\top = h_1 \left(A + \int_a^x g_1^\top h_2 dx \right) \mathcal{D}^{-1} g_2^\top - h_1 \mathcal{D}^{-1} \left(A + \int_a^x g_1^\top h_2 dx \right) g_2^\top .$$

Proof. By direct calculation from the Leibnitz rule (3) for $n = -1$ we obtain:

$$h_1 \mathcal{D}^{-1} g_1^\top h_2 \mathcal{D}^{-1} g_2^\top = h_1 \sum_{i=0}^{\infty} (-1)^i (g_1^\top h_2)^{(i)} \mathcal{D}^{-i-2} g_2^\top ,$$

$$\begin{aligned} & h_1 \left(A + \int_a^x g_1^\top h_2 dx \right) \mathcal{D}^{-1} g_2^\top - h_1 \sum_{i=0}^{\infty} (-1)^i \left(A + \int_a^x g_1^\top h_2 dx \right)^{(i)} \mathcal{D}^{-i-1} g_2^\top \\ & \quad = -h_1 \sum_{i=1}^{\infty} (-1)^i \left(A + \int_a^x g_1^\top h_2 dx \right)^{(i)} \mathcal{D}^{-i-1} g_2^\top = h_1 \sum_{i=0}^{\infty} (-1)^i (g_1^\top h_2)^{(i)} \mathcal{D}^{-i-2} g_2^\top . \end{aligned}$$ ■

Lemma 3. *Let $C^* = -C = \mathrm{const} \in \mathrm{Mat}_{K \times K}(\mathbb{C})$, $\varphi = \varphi(x)$ be a matrix $(N \times K)$ function and $\varphi \in L_2(-\infty, s)$ $\forall\, s \in \mathbb{R}$. Then $w_0^{-1} = w_0^*$, where*

$$w_0 := I - \varphi \left(C + \int_{-\infty}^x \varphi^* \varphi_x dx \right)^{-1} \varphi^* := I - \varphi \Omega^{-1} \varphi^* . \tag{4}$$

Proof.

$$w_0^* = I - \varphi \Omega^{*-1} \varphi^* = I - \varphi \left(\varphi^* \varphi - C - \int\limits_{-\infty}^{x} \varphi^* \varphi_x dx \right)^{-1} \varphi^* , \tag{5}$$

$$\begin{aligned} w_0 w_0^* = I - \varphi & \left[\left(C + \int_{-\infty}^x \varphi^* \varphi_x dx \right)^{-1} + \left(\varphi^* \varphi - C - \int_{-\infty}^x \varphi^* \varphi_x dx \right)^{-1} \right. \\ & \left. - \left(C + \int\limits_{-\infty}^{x} \varphi^* \varphi_x dx \right)^{-1} \varphi^* \varphi \left(\varphi^* \varphi - C - \int\limits_{-\infty}^{x} \varphi^* \varphi_x dx \right)^{-1} \right] \varphi^* \\ = I - \varphi \Omega^{-1} & \left[I + (\Omega - \varphi^* \varphi) \Omega^{*-1} \right] \varphi^* = I . \end{aligned}$$ ■

Theorem 1. *Let $W := w_0 + \varphi \Omega^{-1} \mathcal{D}^{-1} \varphi_x^*$ (see conditions of Lemma 3). Then $W^{-1} = \mathcal{D}^{-1} W^* \mathcal{D}$ (i.e. W is a D-unital operator).*

Proof.

1. $W = I - \varphi \Omega^{-1} \varphi^* + \varphi \Omega^{-1} \mathcal{D}^{-1} \varphi_x^* = I - \varphi \Omega^{-1} \mathcal{D}^{-1} \varphi^* \mathcal{D}$,
   $W^* = I - \mathcal{D} \varphi \mathcal{D}^{-1} \Omega^{*-1} \varphi^*$,
   $\mathcal{D}^{-1} W^* \mathcal{D} = I - \varphi \mathcal{D}^{-1} \Omega^{*-1} \varphi^* \mathcal{D} = w_0^{-1} + \varphi \mathcal{D}^{-1} \left(\Omega^{*-1} \varphi^* \right)_x$.

2. $W\mathcal{D}^{-1}W^*\mathcal{D} = \left(w_0 + \varphi\Omega^{-1}\mathcal{D}^{-1}\varphi_x^*\right)\left(w_0^{-1} + \varphi\mathcal{D}^{-1}(\Omega^{*-1}\varphi^*)_x\right)$

$$= I + w_0\varphi\mathcal{D}^{-1}\left(\Omega^{*-1}\varphi^*\right)_x + \varphi\Omega^{-1}\mathcal{D}^{-1}\varphi_x^*w_0^{-1} + \varphi\Omega^{-1}\mathcal{D}^{-1}\varphi_x^*\varphi\mathcal{D}^{-1}\left(\Omega^{*-1}\varphi^*\right)_x, \quad (6)$$

and using Lemma 2, the definitions of Ω and Ω^* (4)–(5) by direct calculation we obtain that the sum of integral operators in (6) is equal to zero, i.e. $W\mathcal{D}^{-1}W^*\mathcal{D} = I$. ■

3 Lax equation invariant under reductions of $\mathcal{D}$-Hermitian conjugation

In this paper we restrict ourselves by the scalar cases ($N = 1$) of the algebra (2).

We consider the modified Kadomtsev–Petviashvili (mKP) hierarchy [2]

$$\alpha_n \frac{\partial Z}{\partial t_n} = -\left(Z\mathcal{D}^n Z^{-1}\right)_{<1} Z, \qquad \alpha_n \in \mathbb{C}, \quad n \in \mathbb{N}, \quad t_1 := x, \tag{7}$$

where integral operator Z is given by

$$\zeta_{<1} \ni Z = z_0 + z_1\mathcal{D}^{-1} + z_2\mathcal{D}^{-2} + \cdots \quad \left(z_0^{-1} \text{ exists}\right). \tag{8}$$

With the use of the MDO $L := Z\mathcal{D}Z^{-1} := L_{\mathrm{mKP}} = \mathcal{D} + U_0 + U_1\mathcal{D}^{-1} + U_2\mathcal{D}^{-2} + \cdots$, system (7) can be rewritten in the form of the Lax representation

$$\alpha_m L_{t_m} = [B_m, L] := B_m L - LB_m, \tag{9}$$

where $B_m := (L^m)_{>0}$, $m \in \mathbb{N}$.

The mKP hierarchy (7) can be transformed into Zakharov–Schabat equations

$$\alpha_n B_{m_{t_n}} - \alpha_m B_{n_{t_m}} + [B_m, B_n] = 0, \qquad m, n \in \mathbb{N}, \quad \alpha_m, \alpha_n \in \mathbb{C}. \tag{10}$$

Note that the subscripts mean partial differentiations with respect to the indicated variables (evolutionary parameters t_j, $j \in \mathbb{N}$). If we eliminate $U_0, U_1, U_2, \ldots$ from (9), the remaining equations for the function $U := U_0$ in (9) (or in (10)) for $t_1 := x$, $t_2 := y$, $t_3 := t$ would represent the mKP equation

$$\alpha_3 U_t = \frac{1}{4}U_{xxx} - \frac{3}{2}U^2 U_x + \frac{3}{4}\alpha_2^2 \partial_x^{-1} U_{yy} + \frac{3}{2}\alpha_2 U_x \partial_x^{-1} U_y, \tag{11}$$

where $\partial_x^{-1} f := \int^x f dx$, and its hierarchy flows.

W. Oevel and W. Strampp have also introduced so-called constrained modified Kadomtsev–Petviashvili (cmKP) [3], apart from the cKP (constrained KP) hierarchy [4, 5, 6, 7] (see, also [8]). The Lax operator of the cmKP hierarchy is defined by

$$L_{\mathrm{cmKP}} = \mathcal{D}^n + u_{n-1}\mathcal{D}^{n-1} + \cdots + u_1\mathcal{D} + u_0 + \mathcal{D}^{-1}s, \tag{12}$$

or

$$L_{\mathrm{cmKP}} := (L^n_{\mathrm{mKP}})_{\geq 0} + \mathcal{D}^{-1}s,$$

and the hierarchy flows are described by

$$\alpha_m \frac{\partial L_{\mathrm{cmKP}}}{\partial t_m} = \left[\left(L^{m/n}_{\mathrm{cmKP}}\right)_{>0}, \ L_{\mathrm{cmKP}}\right], \qquad \alpha_m s_{t_m} = -\left(L^{m/n}_{\mathrm{cmKP}}\right)^*_{>0}(s). \tag{13}$$

We proposed another restriction of mKP hierarchy, so-called $\mathcal{D}$-Hermitian cmKP ($\mathcal{D}$HcmKP) hierarchy in the form

$$L_{\mathcal{D}\mathrm{HcmKP}} := L_n = \mathcal{D}^n + u_{n-1}\mathcal{D}^{n-1} + \cdots + u_1\mathcal{D} - V, \tag{14}$$

where $\zeta_{<1} \ni V$ is $\mathcal{D}$-Hermitian ($\mathcal{D}$-skew-Hermitian) integral degenerated Volterra operator, defined as

$$V = \boldsymbol{q}\mathcal{M}\mathcal{D}^{-1}\boldsymbol{q}^*\mathcal{D} = (\boldsymbol{q}\mathcal{M}\boldsymbol{q}^*) - \boldsymbol{q}\mathcal{M}\mathcal{D}^{-1}\boldsymbol{q}^*_x, \qquad \text{if} \quad n = 2k,$$

where $\mathcal{M}^* = \mathcal{M}$,(or $V = i\boldsymbol{q}\mathcal{M}\mathcal{D}^{-1}\boldsymbol{q}^*\mathcal{D}$, if $n = 2k-1$); $\boldsymbol{q} = (q_1, \ldots, q_l)$, $k, l \in \mathbb{N}$, and additional reduction for operator L_n:

$$L_n^* = \mu\mathcal{D}L_n\mathcal{D}^{-1}, \qquad \mu = \pm 1.$$

In this case, the $\mathcal{D}$-unital operator $Z := W$ (the definition of integral operator W see below in Theorem 1) is the transformation (dressing) operator for mKP hierarchy (7)–(10). We now work out a few examples of restrictions of the mKP hierarchy connected with $\mathcal{D}$-Hermitian Lax operators of the form (14). We consider the evolution equations

$$\alpha_m L_{n_{t_m}} = [B_m, L_n], \tag{15}$$

where $L_n := L_{\mathcal{D}\mathrm{HcmKP}}$ (14), and B_m are fractional powers m/n of L_n; $n, m, \in \mathbb{N}$. The "basic root" $L_n^{\frac{1}{n}} = \mathcal{D} + a_0 + a_{-1}\mathcal{D}^{-1} + \cdots$ is calculated by requiring $(L_n^{\frac{1}{n}})^n = L_{\mathcal{D}\mathrm{HcmKP}}$. This leads to straightforward recursive scheme for the coefficients $a_0, a_{-1}, \ldots$ of $L_n^{\frac{1}{n}}$, from which these coefficients can be calculated as differential expressions of $u_{n-1}, u_{n-2}, \ldots, u_1, \boldsymbol{q}, \boldsymbol{q}^*$. Higher fractional powers $L_n^{m/n}$ of L_n are then calculated as powers $L_n^{m/n} = (L_n^{1/n})^m$ of this "basic root". By construction, the first question with $m = 1$ in the hierarchy (14) is given by $L_{n_{t_1}} = [\mathcal{D}, L_n] = \frac{\partial L_n}{\partial x}$, so that the first time variable t_1 may be identified with the underlying space variable x.

4 Some examples of equations from the $\mathcal{D}$HcmKP flow

Let us $n = 1$. For $L_1 = \mathcal{D} - i\boldsymbol{q}\mathcal{M}\mathcal{D}^{-1}\boldsymbol{q}^*\mathcal{D}$ the first nontrivial equations in (15) are given by ($\alpha_2 = i$)

$$i\boldsymbol{q}_{t_2} = \boldsymbol{q}_{xx} - 2i\boldsymbol{q}\mathcal{M}\boldsymbol{q}^*\boldsymbol{q}_x, \tag{16}$$

which are the first equations in the multicomponent modified nonlinear Schrödinger hierarchy discussed in [9].

$n = 2$. For $L_2 = \mathcal{D}^2 + iu\mathcal{D} - \boldsymbol{q}\mathcal{M}\mathcal{D}^{-1}\boldsymbol{q}^*\mathcal{D}$ we obtain

$$i\boldsymbol{q}_{t_2} = \boldsymbol{q}_{xx} + iu\boldsymbol{q}_x, \qquad u_{t_2} = 2(\boldsymbol{q}\mathcal{M}\boldsymbol{q}^*)_x, \qquad \alpha_2 = i, \tag{17}$$

$$\boldsymbol{q}_{t_3} = \boldsymbol{q}_{xxx} + \frac{3}{2}iu\boldsymbol{q}_{xx} - \left(\frac{3}{8}u^2 + \frac{3}{2}\boldsymbol{q}\mathcal{M}\boldsymbol{q}^* - \frac{3}{4}iu_x\right)\boldsymbol{q}_x,$$

$$u_{t_3} = \frac{1}{4}u_{xxx} + \frac{3}{8}u^2u_x - \frac{3}{2}\left(\boldsymbol{q}\mathcal{M}\boldsymbol{q}^*u\right)_x, \qquad \alpha_3 = 1. \tag{18}$$

This represents the modified KdV hierarchy coupled with its eigenfunctions. The system (17) is the new multicomponent integrable model of Yajima–Oikawa type [9, 10]. The next higher flow in this hierarchy has the following form ($\alpha_4 = -i$):

$$u_{t_4} = 2\left[\boldsymbol{q}\mathcal{M}\boldsymbol{q}^*_{xx} + \boldsymbol{q}_{xx}M\boldsymbol{q}^* + (\boldsymbol{q}\mathcal{M}\boldsymbol{q}^*)^2\right]_x, \qquad i\boldsymbol{q}_{t_4} = \left(L_2^2\right)_{>0}(\boldsymbol{q}). \tag{19}$$

5 Method of integration of the Lax equation from the $\mathcal{D}$HcmKP hierarchy

There are many mathematical and physical problems associated with the $\mathcal{D}$HcmKP hierarchy. However, the most important one, may be, is finding the soliton solutions for the equations from this hierarchy. We have shown (see Lemma 1) that the $\mathcal{D}$-unital M$\mathcal{D}$O W transforms $\mathcal{D}$-Hermitian operator L into $\mathcal{D}$-Hermitian operator $\hat{L}$ by the dressing transformation $L \to WLW^{-1} := \hat{L}$. Now, we want to extend the previous results to the equations from the $\mathcal{D}$HcmKP hierarchy.

Theorem 2. *Let $\varphi = (\varphi_1, \dots, \varphi_K)$, $K \in \mathbb{N}$ be a smooth fast decreasing on the $-\infty$ complex value K-component vector-function of variable $x \in \mathbb{R}$ and an evolution parameter $t_2 \in \mathbb{R}$ which satisfy additional conditions:*

a) φ be a solution of the equation $i\varphi_{t_2} = \varphi_{xx}$,

b) $\varphi_x = \varphi\Lambda$, where $\Lambda = \operatorname{diag}(\lambda_1, \lambda_2, \dots, \lambda_K) = \mathrm{const}$; $\lambda_j := \lambda_{j1} + i\lambda_{j2} \in \mathbb{C}$; $\lambda_{j1} > 0$, $j = \overline{1, K}$.

Then the vector-function

$$\boldsymbol{q} := \varphi\Omega^{-1} = \varphi\left(C + \int_{-\infty}^{x} \varphi^*\varphi dx\Lambda\right)^{-1} \tag{20}$$

is a solution of the mNSE (16) with the matrix $\mathcal{M} = -i(C\Lambda + \Lambda^* C)$, where $C^* = -C$ is a skew-Hermitian $(K \times K)$ complex matrix.

Proof. The proof is constructed by direct calculation. Using the lemmas we get

$$\begin{aligned} &L_0 := \mathcal{D} \to L := W\mathcal{D}W^{-1} = \mathcal{D} - \varphi\Omega^{-1}(C\Lambda - \Lambda^* C^*)\mathcal{D}^{-1}\Omega^{*-1}\varphi^*\mathcal{D}, \\ &M_0 := i\partial_{t_2} - \mathcal{D}^2 \to M := WM_0W^{-1} = \left(L^2\right)_{>0}, \end{aligned} \tag{21}$$

and from the trivial equation $[L_0, M_0] = 0$ we obtain that $[L, M] = 0$. ■

Corollary 1. *Let $K \geq l \in \mathbb{N}$ and matrix $C = \frac{i}{2}\operatorname{diag}\left(\frac{\mu_1}{\lambda_{11}}, \dots, \frac{\mu_l}{\lambda_{l1}}, 0, \dots, 0\right) \in \mathrm{Mat}_{K\times K}(i\mathbb{R})$.*

Then the function $\boldsymbol{q} = (q_1, \dots, q_l)(x, t_2)$, where $\boldsymbol{q} := \varphi\Omega^{-1}$ and

$$q_j = (-1)^{K-j}\frac{\begin{vmatrix}\Omega_{(j)} \\ \varphi\end{vmatrix}}{\Omega}, \qquad j = 1, \dots, l \tag{22}$$

is a solution of the l-component mNSE

$$i\boldsymbol{q}_{t_2} = \boldsymbol{q}_{xx} - 2i\sum_{j=1}^{l}\mu_j|q_j|^2\boldsymbol{q}_x. \tag{23}$$

Here $\Omega_{(j)}$ is obtained from Ω by deletion of j-line and $|\Omega| := \det\Omega$. In order to prove (22) we use the well-known algebraic equality for framed determinant

$$\det\begin{pmatrix}\Omega & \varphi^* \\ \varphi & \alpha\end{pmatrix} := \begin{vmatrix}\Omega & \varphi^* \\ \varphi & \alpha\end{vmatrix} = \alpha\det\Omega - \varphi\Omega^c\varphi^*, \qquad \alpha \in \mathbb{C}, \tag{24}$$

where Ω^c is the matrix of cofactors, and then

$$q_j = \varphi\Omega^{-1}e_j^T, \qquad e_k := (e_{k_1}, e_{k_2}, \dots, e_{k_K}), \qquad e_{k_m} = \delta_k^m.$$

For $l = 1$ we obtain from the formula (22) the K-soliton solution for the scalar mNSE [11, 12]

$$q = (-1)^{K+1} \frac{\begin{vmatrix} \Omega_{(1)} \\ \varphi \end{vmatrix}}{|\Omega|}, \qquad \Omega = (w_{mn}), \qquad m, n = \overline{1, K},$$

$$\varphi_m = \varphi_{m_0} \exp\left\{\lambda_m x - i\lambda_m^2 t_2\right\}, \qquad \varphi_{m_0} = \mathrm{const},$$

$$w_{mn} = \frac{i\mu}{2\lambda_{11}}\delta_1^{mn} + \frac{\lambda_n}{\bar{\lambda}_m + \lambda_n}\bar{\varphi}_{m0}\varphi_{n0} \exp\left\{\left(\bar{\lambda}_m + \lambda_n\right) x + i\left(\bar{\lambda}_m^2 - \lambda_n^2\right) t_2\right\},$$

$$iq_{t_2} = q_{xx} - 2i\mu|q|^2 q_x. \tag{25}$$

In particular, if $K = 1$, then the previous formulas represent a one-soliton solution for the (25):

$$q = \frac{2\lambda_{11}\varphi_0 \exp\{\lambda x - i\lambda^2 t_2\}}{i\mu + |\varphi_0|^2 \exp\{2\lambda_{11} x + 4\lambda_{11}\lambda_{12} t_2\}},$$

where $\varphi_0 := \varphi_{10}$, $\lambda := \lambda_1 = \lambda_{11} + i\lambda_{12} := \mathrm{Re}\,\lambda + i\,\mathrm{Im}\,\lambda$.

6 Conclusion

In conclusion, we hope that the method of integration of Lax equations with $\mathcal{D}$-Hermitian reductions presented here will be generalized to other nonlinear models from the $\mathcal{D}$HcmKP hierarchy (and in the matrix case too). Similar generalizations for Hermitian reductions in cKP hierarchy [7, 8] were considered in the papers [13, 14], but these results were obtained using the methods from the article [15].

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