Point Form Relativistic Quantum Mechanics and an Algebraic Formulation of Electron Scattering

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Electron scattering off hadronic systems is used to motivate an algebraic approach to hadronic physics. Point form relativistic quantum mechanics, in which all interactions are in the four-momentum operator, along with current operators, is shown to form an infinite dimensional algebra, the representations of which would then generate the observables in electron scattering, namely form factors and structure functions. Several examples of such algebras are given.

1 Electron scattering and point form relativistic quantum mechanics

Electron scattering provides an important tool for investigating the structure of hadrons, both at the nuclear and quark levels. The cross sections measured by experimentalists, for example in inclusive scattering, are exhibited in such a way that they indicate the degree to which an object being probed is not a point particle. The measure of an object not being pointlike is given by sums of squares of form factors. It is well known that the number of independent form factors in elastic scattering is equal to $2s + 1$, where s is the spin of the object. Associated with each of the $2s + 1$ form factors is a static property of the object, such as its charge or magnetic moment, obtained by evaluating the appropriate form factor at zero momentum transfer, $Q^2 = 0$. If every form factor for a spin s object were a constant as a function of Q^2, the object would be a genuine point particle. If form factors are not constant as functions of Q^2, the object has an internal structure, which is revealed in electron scattering experiments as deviations from pointlike cross sections.

Form factors are matrix elements of electromagnetic current operators and provide the link to electron scattering experiments. In order to compute such form factors it is necessary to know both how the objects are described in terms of their constituents, and the nature of the currents of the constituents. Two examples of form factor calculations will be given in the following two sections, one the elastic deuteron form factor in terms of proton and neutron constituents, the other nucleon form factors with three quark constituents.

The goal of this paper is to present an algebraic formulation of electron scattering, algebraic in the sense that the operators describing hadronic dynamics and currents close under commutation. The context for such a formulation is point form relativistic quantum mechanics [1], in which all of the hadronic dynamics is put into the four-momentum operator and the Lorentz generators are all kinematic.

It is then convenient to write the Poincaré commutation relations, necessary for the theory to be properly relativistic, not in terms of the ten generators, but rather in terms of the four-momentum operators that contain the interactions, and global Lorentz transformations:

$$[P_\mu, \ P_\nu] = 0, \tag{1}$$
$$U_\Lambda P_\mu U_\Lambda^{-1} = \left(\Lambda^{-1}\right)_\mu^{\ \nu} P_\nu, \tag{2}$$

where U_Λ is a unitary operator representing the Lorentz transformation Λ. These rewritten Poincaré relations will be called the point form equations, and are the fundamental equations that have to be satisfied for the system of interest. The mass operator is given by $M = \sqrt{P \cdot P}$ and must have a spectrum that is bounded from below.

The simplest example of the point form equations is given by the irreducible representations of the Poincaré group for a single particle of mass m and spin j. If $|p, \sigma\rangle$ is an eigenstate of four-momentum p (with $p \cdot p = m^2$) and spin projection σ, then

$$P_\mu |p, \sigma\rangle = p_\mu |p, \sigma\rangle, \tag{3}$$

$$U_\Lambda |p, \sigma\rangle = \sum |\Lambda p, \sigma'\rangle D^j_{\sigma', \sigma}(R_W), \tag{4}$$

with R_W a Wigner rotation defined by $R_W = B^{-1}(\Lambda v)\Lambda B(v)$, and $B(v)$ a canonical spin (rotationless) boost (see reference [2]) with argument $v = p/m$. Many-particle operators with the same transformation properties as the single particle ones are conveniently obtained by introducing creation and annihilation operators. Let $a^\dagger(p, \sigma)$ be the operator that creates the state $|p, \sigma\rangle$ from the vacuum. If $a(p, \sigma)$ is its adjoint, these operators must satisfy the following relations:

$$[a(p, \sigma), a^\dagger(p', \sigma')]_\pm = E\delta^3(p - p')\delta_{\sigma, \sigma'}, \tag{5}$$

$$U_a a^\dagger(p, \sigma)U_a^{-1} = e^{ip \cdot a}a^\dagger(p, \sigma), \tag{6}$$

$$P_\mu(\text{fr}) = \sum \int \frac{d^3 p}{E} p_\mu a^\dagger(p, \sigma)a(p, \sigma), \tag{7}$$

$$U_\Lambda a^\dagger(p, \sigma)U_\Lambda^{-1} = \sum a^\dagger(\Lambda p, \sigma')D^j_{\sigma', \sigma}(R_W). \tag{8}$$

Here $P_\mu(\text{fr})$ is the free four-momentum operator and plays a role analogous to the free Hamiltonian in nonrelativistic quantum mechanics. Again it is straightforward to show that P_μ satisfies the point form equations. U_a in equation (6) is the unitary operator representing the four-translation a.

To prepare for the construction of interacting four-momentum operators, out of which the interacting mass operators will be built, it is convenient to introduce velocity states, states with simple Lorentz transformation properties. If a Lorentz transformation is applied to a many-particle state, $|p_1, \sigma_1, \ldots, p_n, \sigma_n\rangle = a^\dagger(p_1, \sigma_1) \cdots a^\dagger(p_n, \sigma_n)|0\rangle$, then it is not possible to couple all the momenta and spins together to form spin or orbital angular momentum states, because the Wigner rotations associated with each momentum are different. However, velocity states, defined as n-particle states in their overall rest frame boosted to a four-velocity v will have the desired Lorentz transformation properties:

$$|v, \vec{k_i}, \mu_i\rangle := U_{B(v)}|k_1, \mu_1, \ldots, k_n, \mu_n\rangle \tag{9}$$

$$= \sum |p_1, \sigma_1, \ldots, p_n, \sigma_n\rangle \prod D^{j_i}_{\sigma_i, \mu_i}(R_{W_i}), \tag{10}$$

$$U_\Lambda |v, \vec{k_i}, \mu_i\rangle = U_\Lambda U_{B(v)}|k_1, \mu_1, \ldots, k_n, \mu_n\rangle = U_{B(\Lambda v)}U_{R_W}|k_1, \mu_1, \ldots, k_n, \mu_n\rangle$$

$$= \sum |\Lambda v, R_W \vec{k_i}, \mu_i'\rangle \prod D^{j_i}_{\mu_i', \mu_i}(R_W). \tag{11}$$

Unlike the Lorentz transformation of an n-particle state, where all the Wigner rotations of the D functions are different, in equation (11) it is seen that the Wigner rotations in the D functions are all the same and given by equation (4). Moreover the same Wigner rotation also multiplies the internal momentum vectors, which means that for velocity states, spin and orbital angular momentum can be coupled together exactly as is done nonrelativistically (for more details see reference [2]). The relationship between single particle and internal momenta is given by $p_i = B(v)k_i$, $\sum \vec{k_i} = 0$ and R_{W_i} in equation (10) by replacing p by k_i and Λ by $B(v)$

in equation (4). From the definition of velocity states it then follows that

$$V_\mu |v, \vec{k_i}, \mu_i\rangle = v_\mu |v, \vec{k_i}, \mu_i\rangle, \tag{12}$$

$$M_{fr} |v, \vec{k_i}, \mu_i\rangle = m_n |v, \vec{k_i}, \mu_i\rangle, \tag{13}$$

$$P_\mu(fr) |v, \vec{k_i}, \mu_i\rangle = m_n v^\mu |v, \vec{k_i}, \mu_i\rangle, \tag{14}$$

with $m_n = \sum \sqrt{m_i^2 + \vec{k_i}^2}$ the mass of the n-particle velocity state and $P_\mu(\text{fr}) = M_{\text{fr}} V_\mu$. On velocity states the free four-momentum operator has been written as the product of the four-velocity operator times the free mass operator, which is the so-called Bakamjian–Thomas construction [3] in the point form.

To introduce interactions, write $P_\mu = M V_\mu$, $M = M_{\text{free}} + M_I$. Such a four-momentum operator will satisfy the point form equations if the velocity state kernel, $\langle v', \vec{k_i}', \mu_i' | M_I | v, \vec{k_i}, \mu_i \rangle$ is independent of v and rotationally invariant (which is the same as the nonrelativistic condition on potentials). With such a four-momentum operator, the point form equations become a mass eigenvalue equation:

$$M\Psi = m\Psi, \tag{15}$$

which gives the bound and scattering wavefunctions.

Besides the mass operator, the other quantity needed to compute form factors is a current operator. Current operators must satisfy general properties such as Poincaré covariance and current conservation. In the point form the current operator at the space-time point 0 plays a special role in that it determines the Poincaré covariance and conservation properties at an arbitrary space-time point x. In fact it is easy to see that if $J_\mu(0)$ satisfies

$$U_\Lambda J_\mu(0) U_\Lambda^{-1} = (\Lambda_\mu^\nu)^{-1} J_\nu(0),$$
$$[P^\mu, J_\mu(0)] = 0,$$

then $J_\mu(x) := e^{iP\cdot x} J_\mu(0) e^{-iP\cdot x}$ is Poincaré covariant and is conserved.

Form factors are current operator matrix elements. If the states are chosen to be eigenstates of the four-momentum operator, then the covariance properties of the states and current operators make it possible to greatly simplify the structure of the form factors. As shown in reference [4] current operators are irreducible tensor operators of the Poincaré group, so that a generalized Wigner–Eckart theorem can be used to decompose current matrix elements into Clebsch–Gordan coefficients times reduced matrix elements, which are the invariant form factors. There is a natural frame in which the Clebsch–Gordan coefficients are one, namely the Breit frame, indicated by $p(\text{st})$ (st=standard=Breit) below:

$$\langle p'j'\sigma'I' | J^\mu(0) | pj\sigma I \rangle = \sum \Lambda_\nu^\mu(p',p) D_{\sigma'r'}^{j'}(R_W') F_{r'r}^\nu(Q^2) D_{r\sigma}^j(R_W^{-1}), \tag{16}$$

$$\langle p'(\text{st})j'r'I' | J^\mu(0) | p(\text{st})jrI \rangle = F_{r'r}^\mu(Q^2), \tag{17}$$

$$p'(\text{st}) = m'(\cosh\Delta, 0, 0, \sinh\Delta), \qquad p(\text{st}) = m(\cosh\Delta, 0, 0, -\sinh\Delta),$$

$$Q^2 = (p'(\text{st}) - p(\text{st}))^2 = (m' - m)^2 - 4m'm\sinh\Delta^2,$$

$$p' = \Lambda(p',p)p'(\text{st}), \qquad p = \Lambda(p',p)p(\text{st}).$$

$\Lambda(p',p)$ is a Lorentz transformation that carries the two standard four-momenta to arbitrary four-momenta, while the Wigner rotations in equation (16) are formed from these four-momenta with $\Lambda(p',p)$.

It can then be shown that the invariant form factors in equation (17), indexed by the spin projection labels r' and r, always give the correct number of independent form factors [4]. In fact

$F_{r'r}^{\mu=0}(Q^2)$ is a diagonal matrix giving the electric form factors, $F_{r'r}^{\mu=1,2}(Q^2)$ is an off-diagonal matrix giving the magnetic form factors, and $F_{r'r}^{\mu=3}(Q^2) = 0$ is an expression of current conservation in the Breit frame. To actually compute an invariant form factor using equation (17) a choice for the current operator must be made; usually one begins with a one-body current operator, resulting in what is called the point form spectator approximation (PFSA) [5]. This means that the four-momenta of the unstruck constituents do not change, which has the consequence that the momentum transfer to the struck constituent is greater than the momentum transfer to the object as a whole. As will be seen in the next sections, this has important consequences for the behavior of the form factors as a function of the momentum transfer Q^2.

With the assumption of a one-body current operator, equation (17) can be written more explicitly as

$$F_{r'r}^{\mu}(Q^2) = \sum \int \mathcal{J} d^3\vec{k}_i \mathcal{J}' d^3\vec{k}_i' \Psi_{m'j'r'}^*(\vec{k}_i', \mu_i') u(p_1'\sigma_1')\gamma^\mu u(p_1\sigma_1) F\left((p_1' - p_1)^2\right)$$
$$\times E_{i\neq 1}\delta^3(\vec{p}_i' - \vec{p}_i)\delta_{\sigma'\sigma}\Psi_{mjr}(\vec{k}_i\mu_i), \tag{18}$$

where Ψ is an eigenfunction of the mass operator, $\mathcal{J}$ and $\mathcal{J}'$ are Jacobian factors, and the delta functions express the fact that the momenta of the unstruck constituents do not change. The one-body current matrix element in equation (18) has been chosen for a spin 1/2 particle with form factor F.

2 Elastic deuteron form factors

To compute elastic deuteron form factors using the point form it is necessary to have a mass operator that will generate the deuteron wave functions. To make use of the many nonrelativistic potentials that are able to give good deuteron wave functions, the mass operator, a sum of relativistic kinetic energy and interaction, is squared and then rewritten in the form of a nonrelativistic Schrödinger equation [6]:

$$M = 2\sqrt{m_N^2 + \vec{k}^2} + M_{\text{int}}, \qquad M^2 = 4\left(m_N^2 + \vec{k}^2\right) + 4m_N V_{N-N}, \tag{19}$$

$$M^2\Psi = \left(4m_N^2 + 4\vec{k}^2 + 4m_N V_{N-N}\right)\Psi = m_D^2\Psi,$$

$$\left(\frac{\vec{k}^2}{m_N} + V_{N-N}\right)\Psi = \left(\frac{m_D^2}{4m_n} - m_N\right)\Psi; \tag{20}$$

in this work the Argonne v_{18} and Reid'93 potentials were used to obtain the deuteron wave functions.

Since the nucleons that make up the deuteron themselves have internal structure, it is necessary to choose form factors for them. In this calculation the one-body current operators were determined by form factors given by Gari, Krümpelmann [7] and Mergell, Meissner, and Drechsel [8].

The results of these calculations have been published in reference [5]. Collaborators are T. Allen and W. Polyzou, with much help from F. Coester and G. Payne. A comparison of our results with those of other calculations is given by F. Gross [9], where it is seen that the structure function falls off too fast in comparison with experimental data, while the results for the tensor polarization agree reasonably well with data. These results show the need for including two-body currents in the form factor calculations, a subject which is discussed elsewhere [10].

3   Nucleon form factors

To compute nucleon form factors the mass operator is obtained in a rather different way as compared with the deuteron mass operator. In this case the three quark mass operator comes from a "semi-relativistic" Hamiltonian, the sum of relativistic kinetic energy, linear confinement potential and hyperfine interaction (Goldstone Boson Exchange model [11]):

$$H \longrightarrow M = \sum \sqrt{m^2 + \vec{k_i}^2} + \sum V(\mathrm{conf}) + \sum V(HF).$$

That is, the "semi-relativistic" Hamiltonian can be reinterpreted as a point form mass operator and the eigenfunctions previously calculated can be used to compute form factors. Thus, the bound state problem for three quarks, $M\Psi = m\Psi$, gives the wave functions and a good spectroscopic fit (Glozman, et al [12]). Finally the current operator is a point-like Dirac current with no anomalous magnetic moment.

When these eigenfunctions and current operators are put into equation (18), excellent agreement with data is obtained. The form factor graphs and static properties can be found in reference [13]. Collaborators in this project include S. Boffi, L. Glozman, W. Plessas, M. Radici, and R. Wagenbrunn. It should be noted that form factors for the weak interactions have also been calculated and give excellent agreement with experiment [14].

4   Algebraic formulation of electron scattering

As shown in previous sections the two quantities needed to calculate electron scattering observables in the point form are the hadronic four-momentum operator P_μ, satisfying $P_\mu^\dagger = P_\mu$ and the electromagnetic current operator $J_\mu(0)$. To rewrite these quantities in an algebraic form it is more convenient to work with the Fourier transform of the current operator

$$J_\mu(Q) = \int d^4x e^{iQ \cdot x} J_\mu(x), \tag{21}$$

with $J_\mu^\dagger(Q) = J_\mu(-Q)$, for then two of the fundamental commutation relations are

$$[P_\mu, P_\nu] = 0, \tag{22}$$
$$[P_\mu, J_\nu(Q)] = Q_\mu J_\nu(Q). \tag{23}$$

These operators must also satisfy Lorentz transformation properties, with the four-momentum operator transforming as a four-vector (equation (2)) and the current operator as a four-vector density. To get an algebraic structure, $[J_\mu(Q), J_\nu(Q')]$ should close. Since equation (22) is a point form equation required by Poincaré covariance, and equation (23) is a consequence of the translational covariance of current operators it is clear that the crucial commutator relation is the one involving the two currents. While the commutator of two currents closing is reminiscent of the current algebra of the 1960's (see for example [15]), the crucial difference is that the currents are not regarded as the fundamental degrees of freedom, to be used in a Hamiltonian; rather in combination with equations (22), (23), the four-momentum operator and the current operator form a closed algebraic system, the representations of which should give the observables of electron scattering.

These observables include the structure tensor for inclusive scattering,

$$W_{\mu\nu}(p, Q) = \sum \int d^4Q' \langle pj\sigma | [J_\mu(Q), J_\nu(Q')] | pj\sigma \rangle, \tag{24}$$

and form factors for exclusive scattering,

$$F_\mu(Q^2) = \langle p'j'\sigma'|J_\mu(Q)|pj\sigma\rangle. \tag{25}$$

Further, it follows from equation (23) that $J_\mu(Q)$ acts as a raising (lowering) operator on eigenstates of P_μ, and for certain values of the momentum transfer Q, acts as an annihilation operator on the ground state:

$$J_\mu(Q)|p_{\text{gnd}}j\sigma\rangle = 0 \tag{26}$$

for $(p_{\text{gnd}} + Q)^2 < m_{\text{gnd}}^2$.

Such an algebraic structure of $\{P_\mu, J_\mu(Q)\}$ is reminiscent of a Cartan algebra, with the diagonal operators and the raising and lowering operators (see for example [16, 17]). Such algebras also have automorphism groups; in the case of the present algebra, the Lorentz transformations are a subgroup of the automorphism group. Finally the annihilation property, equation (26) is analogous to positive energy, or discrete series representations of finite dimensional Lie algebras.

A well known example from two dimensional field theory is the Virasoro algebra [17]:

$$[L_m, L_n] = (m - n)L_{m+n} + \frac{c}{12}m\left(m^2 - 1\right)\delta_{m+n,0}, \tag{27}$$

where $L_0 \approx P_\mu$ is interpreted as a mass or energy and $L_{m\neq 0} \approx J_\mu(Q)$ the space component of a current in discrete variables. There is no analogue of equation (22) unless the product of two Virasoro algebras is used, in which case the interpretation of L_0 becomes the light front operators $P_\pm = P_0 \pm P_1$. But equation (23) is already contained in equation (27) when the index m is set equal to zero. When both m and n are nonzero in equation (27), the commutator gives the closure of two current operators.

As a second example consider a "$U(N)$" model for spinless particles of mass m, with creation and annihilation operators satisfying $[a(p), a^\dagger(p')] = E\delta^3(p - p')$, and out of which the following operators can be built:

$$P_\mu = \int \frac{d^3p}{E}p_\mu a^\dagger(p)a(p), \tag{28}$$

$$J_\mu(x) = \int \frac{d^3p_1}{E_1}\frac{d^3p_2}{E_2}F\left((p_1 - p_2)^2\right)(p_1 + p_2)_\mu e^{i(p_1-p_2)\cdot x}a^\dagger(p_1)a(p_2), \tag{29}$$

$$J_\mu(Q) = F\left(Q^2\right)\int \frac{d^3p_1}{E_1}\frac{d^3p_2}{E_2}\delta^4(p_1 - p_2 - Q)(p_1 + p_2)_\mu a^\dagger(p_1)a(p_2); \tag{30}$$

both the free four-momentum operator (28) and the Fourier transform of the current operator (30) are formed from operators of the form $a_i^\dagger a_j$, which forms the Lie algebra of the unitary group, hence the name "$U(N)$" model. From the definition given of these operators, it can now be shown by direct calculation that equations (22), (23) hold, for an arbitrary form factor $F\left(Q^2\right)$.

The key equation is the commutator of the two currents. Using equation (30) suggests the following possibility:

$$[J_\mu^\dagger(Q), J_\nu(Q)] = 4F\left(Q^2\right)^2(P_\mu Q_\nu + P_\nu Q_\mu), \tag{31}$$

$$[J_\mu(Q), J_\nu(Q')] = 2\frac{F\left(Q^2\right)F\left(Q'^2\right)}{F\left((Q + Q')^2\right)}\left(Q'_\mu J_\nu\left(Q + Q'\right) - Q_\nu J_\mu\left(Q + Q'\right)\right), \tag{32}$$

for $Q + Q' \neq 0$. Equation (31) is the analogue of the Cartan algebra commutator, where the commutator of a raising operator with its adjoint gives a diagonal operator, while equation (32) is similar to the Virasoro algebra (27), when $m + n \neq 0$. Note also that equation (31) has no central extension, as is the case with the Virasoro algebra.

5   Conclusion

Motivated by the analysis of electron scattering experiments, an algebraic formulation of hadronic systems has been given in the context of point form relativistic quantum mechanics. The point form is one of the forms of relativistic quantum mechanics proposed by Dirac, in which all of the interactions are in the four-momentum operator, and the Lorentz generators are all kinematic (free of interactions). As shown in the introduction the other operator besides the four-momentum operator needed to compute form factors and structure functions that provide the link to experimental data is the electromagnetic current operator. While it suffices to know the current operator at the space-time point zero for computing form factors, to uncover an algebraic structure, it is more useful to consider the Fourier transform of the current operator $J_\mu(Q)$, where the independent variable Q is the four-momentum transfer. From the definition of $J_\mu(Q)$ it follows that it acts as a raising or lowering operator on eigenstates of the four-momentum operator.

The key commutator is between two current operators, and here there is no direct guide from hadronic physics. Two examples of algebraic structures were given in the previous section, but there is much work to be done to find physically interesting examples. One possibility is to work with free hadronic systems and then deform the current commutators to produce interactions.

Both P_μ and $J_\mu(Q)$ have definite transformation properties under Lorentz transformations, which suggests that the Lorentz transformations belong to an automorphism group, just as the symmetric group is the automorphism group for the $U(N)$ algebras.

A final important issue concerns the representations of such algebraic structures, for it is the representations that provide the actual form factors and structure functions. Since for certain values of Q, $J_\mu(Q)$ acts as an annihilation operator on the ground state (see equation (26)), the representations of interest should be "discrete series" types of representations (see for example [16, 17]) and it should be possible to generalize the techniques for generating such representations to those needed here.

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