

# Equation for Particles of Spin $\frac{3}{2}$ with Anomalous Interaction

Alexander GALKIN

*Institute of Mathematics of NAS of Ukraine, 3 Tereshchenkivska Str., 01601 Kyiv-4, Ukraine*  
 E-mail: *galkin@imath.kiev.ua*

We consider tensor-bispinor equation, which describes doublets of particles with spin  $\frac{3}{2}$ , nonzero masses and anomalous interaction with electromagnetic field. Using this equation we find the energy levels of the particle with spin  $\frac{3}{2}$  in constant magnetic field, crossed electric and magnetic fields. We show that it is possible to introduce anomalous interaction in such way that energy levels are real for all values of magnetic field and arbitrary gyromagnetic ratio  $g$ .

## 1 Introduction

After the discovery of the acausality of the Rarita–Schwinger equation [1] various equations have been adopted for the description of spin  $\frac{3}{2}$  particles. But these equations also have different defects. Some of these defects are acausal propagation of solutions, complex energies, incorrect value of the gyromagnetic ratio etc [2, 3, 4].

In the present paper we propose the equation for doublets of massive particles with spin  $\frac{3}{2}$  interacting with external electromagnetic field, which do not have many of defects enumerated above. In our case the wave function of particle with spin  $\frac{3}{2}$  is described by irreducible anti-symmetric tensor-bispinor of rank 2. We generalize results [5, 6, 7] for the case of linear and quadratic anomalous interactions with electromagnetic field. Finally we consider the motion of particle with spin  $\frac{3}{2}$  in crossed magnetic and electric fields.

## 2 Tensor-bispinor equation with minimal interaction in electromagnetic field

Here we describe particles with spin  $\frac{3}{2}$  in terms of irreducible antisymmetric tensor-bispinor  $\Psi^{\mu\nu}$  ( $\Psi^{\mu\nu} = -\Psi^{\nu\mu}$ ). In the case of free particle,  $\Psi^{\mu\nu}$  is a 24-component tensor-bispinor,  $\Psi_{\alpha}^{\mu\nu} = -\Psi_{\alpha}^{\nu\mu}$ , which has two tensorial indexes  $\mu, \nu = 0, 1, 2, 3$  and a spinorial index  $\alpha = 0, 1, 2, 3$ . We suppose  $\Psi_{\alpha}^{\mu\nu}$  satisfies the following covariant condition

$$\gamma_{\mu}\gamma_{\nu}\Psi^{\mu\nu} = 0, \tag{1}$$

where  $\gamma^{\mu}$  are Dirac matrices acting on spinor index  $\alpha$  which is omitted.  $\Psi^{\mu\nu}$  must also satisfy the Dirac equation

$$(\gamma_{\lambda}p^{\lambda} - m)\Psi^{\mu\nu} = 0, \tag{2}$$

where  $p_{\mu} = i\frac{\partial}{\partial x^{\mu}}$ . Commuting  $\gamma_{\mu}\gamma_{\nu}$  and  $(\gamma_{\lambda}p^{\lambda} - m)$  we come to the secondary constraint for  $\Psi^{\mu\nu}$

$$p_{\mu}\gamma_{\nu}\Psi^{\mu\nu} = 0. \tag{3}$$

It is possible to show that conditions (1), (3) reduce the number of independent components of  $\Psi^{\mu\nu}$  to 16. Equations (1)–(3) describe a doublet of particles with spin  $\frac{3}{2}$  and mass  $m$  [8, 9].

In order to introduce interaction with electromagnetic field, we rewrite system (1)–(3) as a single equation

$$\begin{aligned} & (\gamma_\lambda p^\lambda - m)\Psi^{\mu\nu} + \frac{1}{12}(p^\mu\gamma^\nu - p^\nu\gamma^\mu)[\gamma_\rho, \gamma_\sigma]\Psi^{\rho\sigma} \\ & - \frac{1}{12}[\gamma^\mu, \gamma^\nu](p_\rho\gamma_\sigma - p_\sigma\gamma_\rho)\Psi^{\rho\sigma} + \frac{1}{24}[\gamma^\mu, \gamma^\nu]\gamma_\lambda p^\lambda[\gamma_\rho, \gamma_\sigma]\Psi^{\rho\sigma} = 0. \end{aligned} \quad (4)$$

The minimal interaction can be introduced in equation (4) in standard way

$$p_\mu \longrightarrow \pi_\mu = p_\mu - eA_\mu, \quad (5)$$

where  $A_\mu$  is the vector-potential of electromagnetic field. As a result we obtain

$$\begin{aligned} & (\gamma_\lambda \pi^\lambda - m)\Psi^{\mu\nu} + \frac{1}{12}(\pi^\mu\gamma^\nu - \pi^\nu\gamma^\mu)[\gamma_\rho, \gamma_\sigma]\Psi^{\rho\sigma} \\ & - \frac{1}{12}[\gamma^\mu, \gamma^\nu](\pi_\rho\gamma_\sigma - \pi_\sigma\gamma_\rho)\Psi^{\rho\sigma} + \frac{1}{24}[\gamma^\mu, \gamma^\nu]\gamma_\lambda \pi^\lambda[\gamma_\rho, \gamma_\sigma]\Psi^{\rho\sigma} = 0. \end{aligned} \quad (6)$$

Contracting (6) with  $\gamma_\mu\gamma_\nu$  and with  $\pi_\mu\gamma_\nu - \pi_\nu\gamma_\mu$  we come to the constraints

$$\gamma_\mu\gamma_\nu\Psi^{\mu\nu} = 0, \quad \pi_\mu\gamma_\nu\Psi^{\mu\nu} = \frac{ie}{m}(F_{\mu\nu} - \gamma^\lambda\gamma_\nu F_{\mu\lambda})\Psi^{\mu\nu}, \quad (7)$$

where  $F_{\mu\nu}$  is the tensor of electromagnetic field  $F_{\mu\nu} = i(p_\mu A_\nu - p_\nu A_\mu)$ . Substituting (7) in (6) we obtain an equation for  $\Psi^{\mu\nu}$

$$(\gamma_\lambda \pi^\lambda - m)\Psi^{\mu\nu} - \frac{ie}{6m}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)(F_{\rho\lambda} - \gamma^\sigma\gamma_\lambda F_{\rho\sigma})\Psi^{\rho\lambda} = 0. \quad (8)$$

Equation (8) is equivalent to introduced in [7]. It can be shown by using the substitution

$$\Psi^{ab} = \frac{1}{2}\epsilon_{abc}\left(\Phi_c^{(1)} + \Phi_c^{(2)}\right), \quad \Psi^{0c} = \frac{i}{2}\left(\Phi_c^{(2)} - \Phi_c^{(1)}\right), \quad (9)$$

( $a, b, c = 1, 2, 3$ ),  $\Phi_c^{(1)}$  and  $\Phi_c^{(2)}$  are bispinors.

### 3 Anomalous interaction

In this section we generalize equation (6) by adding the terms  $T_{\rho\sigma}^{\mu\nu}\Psi^{\rho\sigma}$  and  $T_{\rho\sigma}^{\mu\nu}T_{\delta\epsilon}^{\rho\sigma}\Psi^{\delta\epsilon}$  [8, 9], which are linear and quadratic in  $F^{\mu\nu}$  correspondingly, i.e. consider both minimal and anomalous interactions [2]

$$\begin{aligned} & (\gamma_\lambda \pi^\lambda - m)\Psi^{\mu\nu} + \frac{1}{12}(\pi^\mu\gamma^\nu - \pi^\nu\gamma^\mu)[\gamma_\rho, \gamma_\sigma]\Psi^{\rho\sigma} - \frac{1}{12}[\gamma_\mu, \gamma_\nu](\pi_\rho\gamma_\sigma - \pi_\sigma\gamma_\rho)\Psi^{\rho\sigma} \\ & + \frac{1}{24}[\gamma_\mu, \gamma_\nu]\gamma_\lambda \pi^\lambda[\gamma_\rho, \gamma_\sigma]\Psi^{\rho\sigma} + T_{\rho\sigma}^{\mu\nu}\Psi^{\rho\sigma} + \tilde{T}_{\rho\sigma}^{\mu\nu}\tilde{T}_{\delta\epsilon}^{\rho\sigma}\Psi^{\delta\epsilon} = 0. \end{aligned} \quad (10)$$

We suppose the following relations are satisfied

$$\gamma_\mu T_{\rho\sigma}^{\mu\nu} = 0, \quad \gamma_\mu \tilde{T}_{\rho\sigma}^{\mu\nu} = 0. \quad (11)$$

It is possible to show [8, 9] that (11) are the necessary and sufficient conditions to obtain consistent equation (3) whose solutions propagate with the velocity less then velocity of light. Using  $F^{\mu\nu}$ ,  $\epsilon^{\mu\nu\rho\sigma}$ ,  $g_{\mu\nu}$  and  $\gamma_\mu$  one can construct the basis antisymmetric tensor-bispinors linear in  $F^{\mu\nu}$ :

$$T_{1\rho\sigma}^{\mu\nu} = F_\rho^\nu\gamma^\mu\gamma_\sigma - F_\rho^\mu\gamma^\nu\gamma_\sigma - F_\sigma^\nu\gamma^\mu\gamma_\rho + F_\sigma^\mu\gamma^\nu\gamma_\rho,$$

$$\begin{aligned}
T_{2\rho\sigma}^{\mu\nu} &= F_\rho^\mu \delta_\sigma^\nu - F_\rho^\nu \delta_\sigma^\mu - F_\sigma^\mu \delta_\rho^\nu + F_\sigma^\nu \delta_\rho^\mu, \\
T_{3\rho\sigma}^{\mu\nu} &= \gamma^\nu \gamma^\lambda F_{\rho\lambda} \delta_\sigma^\mu - \gamma^\mu \gamma^\lambda F_{\rho\lambda} \delta_\sigma^\nu - \gamma^\nu \gamma^\lambda F_{\sigma\lambda} \delta_\rho^\mu + \gamma^\mu \gamma^\lambda F_{\sigma\lambda} \delta_\rho^\nu \\
&\quad + \gamma_\rho \gamma^\lambda F_\lambda^\mu \delta_\sigma^\nu - \gamma_\rho \gamma^\lambda F_\lambda^\nu \delta_\sigma^\mu + \gamma_\sigma \gamma^\lambda F_\lambda^\mu \delta_\rho^\nu - \gamma_\sigma \gamma^\lambda F_\lambda^\nu \delta_\rho^\mu, \\
T_{4\rho\sigma}^{\mu\nu} &= (\delta_\sigma^\mu \delta_\rho^\nu - \delta_\sigma^\nu \delta_\rho^\mu) \gamma_\alpha \gamma_\beta F^{\alpha\beta}, \\
T_{5\rho\sigma}^{\mu\nu} &= \gamma_4 (\tilde{F}_\rho^\mu \delta_\sigma^\nu - \tilde{F}_\rho^\nu \delta_\sigma^\mu - \tilde{F}_\sigma^\mu \delta_\rho^\nu + \tilde{F}_\sigma^\nu \delta_\rho^\mu), \\
T_{6\rho\sigma}^{\mu\nu} &= \gamma_4 (F_\alpha^\mu \epsilon_{\rho\sigma}^{\alpha\nu} - F_\alpha^\nu \epsilon_{\rho\sigma}^{\alpha\mu} + F_{\alpha\rho} \epsilon^{\alpha\mu\nu} - F_{\alpha\sigma} \epsilon^{\alpha\mu\nu}), \\
T_{7\rho\sigma}^{\mu\nu} &= (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu) F_{\rho\sigma} + F^{\mu\nu} (\gamma_\rho \gamma_\sigma - \gamma_\sigma \gamma_\rho), \\
T_{8\rho\sigma}^{\mu\nu} &= \gamma^\nu \gamma^\lambda F_{\rho\lambda} \delta_\sigma^\mu - \gamma^\mu \gamma^\lambda F_{\rho\lambda} \delta_\sigma^\nu - \gamma^\nu \gamma^\lambda F_{\sigma\lambda} \delta_\rho^\mu + \gamma^\mu \gamma^\lambda F_{\sigma\lambda} \delta_\rho^\nu \\
&\quad - \gamma_\rho \gamma^\lambda F_\lambda^\mu \delta_\sigma^\nu + \gamma_\rho \gamma^\lambda F_\lambda^\nu \delta_\sigma^\mu - \gamma_\sigma \gamma^\lambda F_\lambda^\mu \delta_\rho^\nu + \gamma_\sigma \gamma^\lambda F_\lambda^\nu \delta_\rho^\mu, \\
T_{9\rho\sigma}^{\mu\nu} &= \gamma_4 (F_\alpha^\mu \epsilon_{\rho\sigma}^{\alpha\nu} - F_\alpha^\nu \epsilon_{\rho\sigma}^{\alpha\mu} - F_{\alpha\rho} \epsilon^{\alpha\mu\nu} + F_{\alpha\sigma} \epsilon^{\alpha\mu\nu}), \\
T_{10\rho\sigma}^{\mu\nu} &= (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu) F_{\rho\sigma} - F^{\mu\nu} (\gamma_\rho \gamma_\sigma - \gamma_\sigma \gamma_\rho),
\end{aligned}$$

where  $\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon_{\rho\sigma}^{\mu\nu} F^{\rho\sigma}$ .

Then the general form of  $T_{\rho\sigma}^{\mu\nu}$  and  $\tilde{T}_{\rho\sigma}^{\mu\nu}$  is the following

$$T_{\rho\sigma}^{\mu\nu} = \sum_{i=1}^{10} \alpha_i T_i^{\mu\nu}, \quad \tilde{T}_{\rho\sigma}^{\mu\nu} = \sum_{i=1}^{10} \tilde{\alpha}_i \tilde{T}_i^{\mu\nu},$$

where  $\alpha_i, \tilde{\alpha}_i$  are arbitrary constants.

Using (11) and asking for existence of real Lagrangian correspondingly to (3)

$$\begin{aligned}
\mathfrak{S} &= \bar{\Psi}_{\mu\nu} (\gamma_\lambda \pi^\lambda - m) \Psi^{\mu\nu} + \frac{1}{12} \bar{\Psi}_{\mu\nu} (\pi^\mu \gamma^\nu - \pi^\nu \gamma^\mu) [\gamma_\rho, \gamma_\sigma] \Psi^{\rho\sigma} - \frac{1}{12} \bar{\Psi}_{\mu\nu} [\gamma_\mu, \gamma_\nu] \\
&\quad \times (\pi_\rho \gamma_\sigma - \pi_\sigma \gamma_\rho) \Psi^{\rho\sigma} + \frac{1}{24} \bar{\Psi}_{\mu\nu} [\gamma_\mu, \gamma_\nu] \gamma_\lambda \pi^\lambda [\gamma_\rho, \gamma_\sigma] \Psi^{\rho\sigma} + \bar{\Psi}_{\mu\nu} T_{\rho\sigma}^{\mu\nu} \Psi^{\rho\sigma} + \bar{\Psi}_{\mu\nu} \tilde{T}_{\rho\sigma}^{\mu\nu} \tilde{T}_{\delta\epsilon}^{\rho\sigma} \Psi^{\delta\epsilon}
\end{aligned}$$

we come to the conditions

$$\alpha_1 = \alpha_6 = -\alpha_9 = \frac{\lambda}{2}, \quad \alpha_2 = 2\lambda, \quad \alpha_3 = \alpha_8 = \frac{\lambda}{4}, \quad \alpha_4 = \alpha_5 = \alpha_7 = \alpha_{10} = 0,$$

where  $\lambda$  is an arbitrary constant. The analogous relations are valid for  $\tilde{\alpha}_i$ .

Substituting (9) into (3) we can express (3) in the Dirac-like form [7]

$$\left( \Gamma_\mu \pi^\mu - m + \frac{e}{4m} (1 - i\Gamma_4) \left( \left( \frac{i}{4} (g-2) [\Gamma_\mu, \Gamma_\nu] + g\tau_{\mu\nu} \right) F^{\mu\nu} + g_1 (S_{\mu\nu} F^{\mu\nu})^2 \right) \right) \Psi^{(1)} = 0, \quad (12)$$

$$\left( \Gamma_\mu \pi^\mu - m + \frac{e}{4m} (1 + i\Gamma_4) \left( \left( \frac{i}{4} (g-2) [\Gamma_\mu, \Gamma_\nu] + g\tau_{\mu\nu} \right) F^{\mu\nu} + g_1 (S_{\mu\nu} F^{\mu\nu})^2 \right) \right) \Psi^{(2)} = 0, \quad (13)$$

where  $g = \frac{2}{3} (1 - \frac{\lambda}{2})$ ,  $g_1 = \frac{\tilde{\lambda}^2}{9}$ ,  $\Psi^{(1)} = (\Phi_1^{(1)}, \Phi_2^{(1)}, \Phi_3^{(1)})^T$ ,  $\Psi^{(2)} = (\Phi_1^{(2)}, \Phi_2^{(2)}, \Phi_3^{(2)})^T$ ,  $S_{\mu\nu} = \frac{i}{4} [\Gamma_\mu, \Gamma_\nu] + \tau_{\mu\nu}$  and  $\tau_{\mu\nu}$  satisfy the relations  $\tau_{ab} = \epsilon_{abc} \tau_c$ ;  $\tau_0 a = i\tau_a$ ,  $\tau_a \tau_a = \tau(\tau + 1)$ ;  $[\tau_a, \tau_b] = i\epsilon_{abc} \tau_c$ ,  $a, b, c = 1, 2, 3$ .

Matrices  $\Gamma_\mu$  and  $\tau_a$  can be represented in the following forms;  $\Gamma_\mu = \gamma_\mu \otimes I_3$ ,  $\tau_a = I_4 \otimes \hat{\tau}_a$ , symbol  $\otimes$  denotes the direct product of matrices,  $\hat{\tau}_a$  are  $3 \times 3$  matrices, realizing the representation  $D(3)$  of the algebra  $AO(3)$ ,  $I_3$  and  $I_4$  are the unit  $3 \times 3$  and  $4 \times 4$  matrices correspondingly.

In the representation (9) constraints (7) are reduced to the forms

$$[(\Gamma_\mu \pi^\mu + m)(1 + i\Gamma_4)(S_{\mu\nu} S^{\mu\nu} - 3)] \Psi^{(1)} = 24m \Psi^{(1)}, \quad (14)$$

$$[(\Gamma_\mu \pi^\mu + m)(1 - i\Gamma_4)(S_{\mu\nu} S^{\mu\nu} - 3)] \Psi^{(2)} = 24m \Psi^{(2)}. \quad (15)$$

We see that the value  $g = 2$  corresponds to the most simple form of equations (12)–(13). Moreover, using Foldy–Wouthysen transformation [7] it can be shown that the Hamiltonian of the equation (12) or (13) in quasiclassical approximation is Hermitian, when  $g = 2$ .

#### 4 Particle with spin $\frac{3}{2}$ in homogeneous magnetic field

Now let us use proposed equations to solve the problem of motion of charged particle with spin  $\frac{3}{2}$  in constant magnetic field.

We start with equation (12) which can be written in the following equivalent form

$$\left(\pi_\mu \pi^\mu - m^2 + \frac{eg}{2} S_{\mu\nu} F^{\mu\nu} + \frac{eg_1}{2} (S_{\mu\nu} F^{\mu\nu})^2\right) \Psi_+^{(1)} = 0, \quad (16)$$

$$(S_{\mu\nu} S^{\mu\nu} - 15) \Psi_+^{(1)} = 0, \quad (17)$$

$$\Psi_-^{(1)} = \frac{1}{m} \Gamma_\mu \pi^\mu \Psi_+^{(1)}, \quad \Psi^{(1)} = \Psi_+^{(1)} + \Psi_-^{(1)} \quad (18)$$

(the similar equation can be obtained for (13)).

The tensor  $F^{\mu\nu}$  corresponding to the constant and homogeneous magnetic field can be chosen in the form

$$F_{0a} = F_{23} = -F_{32} = F_{31} = -F_{13} = 0, \quad a = 1, 2, 3, \quad F_{12} = -F_{21} = H_3 = H, \quad H \geq 0,$$

where  $H$  is the strength of the magnetic field.

The solution of equation (17), (18) is of the form

$$\Psi_+^{(1)} = \begin{pmatrix} \Phi_{\frac{3}{2}}^{(1)} \\ \hat{0} \\ \frac{1}{m} \left(\varepsilon + \frac{2}{3} S_a \pi_a\right) \Phi_{\frac{3}{2}}^{(1)} \\ -\frac{2}{3m} K_a^{\frac{3}{2}} \pi_a \Phi_{\frac{3}{2}}^{(1)} \end{pmatrix}, \quad (19)$$

where

$$\left(K_{\frac{3}{2}}^{\frac{3}{2}}\right)_{mm'} = \delta_{mm'} \sqrt{\frac{9}{4} - m^2}; \quad \left(K_1^{\frac{3}{2}}\right)_{mm'} \pm i \left(K_2^{\frac{3}{2}}\right)_{mm'} = \pm \delta_{m\pm m'} \sqrt{\frac{3}{2} \mp m(m \mp 1) \pm 3m},$$

$m, m' = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$ ;  $\hat{0} = (0, 0)^T$ ,  $\Phi_{\frac{3}{2}}^{(1)}$  is a 4 component spinor which satisfies the equation

$$\left[p^2 + e^2 H^2 x_2^2 - eH (gS_3 + 2g_1 S_3^2 H + 2x_2 p_1)\right] \Phi_{\frac{3}{2}}^{(1)} = (\varepsilon^2 - m^2) \Phi_{\frac{3}{2}}^{(1)}. \quad (20)$$

So the problem of describing the motion of particle with spin  $\frac{3}{2}$  reduces to the solution of equation (20).

Using the eigenvectors  $\Omega_\nu^{\frac{3}{2}}$  of matrix  $S_3$  ( $\nu = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$  are eigenvalues of  $S_3$ ) we can represent  $\Phi_{\frac{3}{2}}^{(1)}$  in the form

$$\Phi_{\frac{3}{2}}^{(1)} = \exp(ip_1 x_1 + ip_3 x_3) \sum_{\nu=-\frac{3}{2}}^{\frac{3}{2}} f_\nu^{\frac{3}{2}}(x_2) \Omega_\nu^{\frac{3}{2}}, \quad (21)$$

here  $f_\nu^{\frac{3}{2}}(x_2)$  are unknown functions,  $\Omega_\nu^{\frac{3}{2}}$  are 4 components spinor eigenvectors of  $S_3$  and  $\Omega_\nu^{\frac{3}{2}}$  are 4 components spinors

$$\Omega_{-\frac{3}{2}}^{\frac{3}{2}} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \Omega_{-\frac{1}{2}}^{\frac{3}{2}} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \Omega_{\frac{1}{2}}^{\frac{3}{2}} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \Omega_{\frac{3}{2}}^{\frac{3}{2}} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

Substituting (21) into (20) we come to the equation for  $f_\nu^{\frac{3}{2}}$

$$\left(-\frac{d^2}{dy^2} + y^2\right) f_\nu^{\frac{3}{2}}(y) = \eta f_\nu^{\frac{3}{2}}(y),$$

where  $y = \frac{1}{\sqrt{eH}}(eHx_2 - p_1)$  and  $\eta = \frac{1}{eH}(\varepsilon^2 - m^2 - p_3^2 + eH(\nu g + 2g_1\nu H))$ . It is the equation for the harmonics oscillator. So, requiring that function  $f_\nu^{\frac{3}{2}} \rightarrow 0$  when  $x_2 \rightarrow \pm\infty$ , we obtain the condition for  $\eta$

$$\eta = 2n + 1, \quad n = 0, 1, 2, 3, \dots,$$

then the energy levels for the particle with spin  $\frac{3}{2}$  in constant magnetic field can be written in the following form

$$\varepsilon^2 = m^2 + p_3^2 + eH(2n + 1 - \nu(g + 2g_1\nu H)), \quad n = 0, 1, 2, 3, \dots \quad (22)$$

Function  $f_\nu^{\frac{3}{2}}$  has the form

$$f_\nu^{\frac{3}{2}}(x_2) = \exp\left(-\frac{eHx_2 - p_1}{2eH}\right) h_n\left(\frac{eHx_2 - p_1}{\sqrt{eH}}\right),$$

where  $h_n(y) = \frac{H_n(y)}{\|H_n(y)\|}$ ,  $H_n(y)$  are Hermitian polynomials.

We note that for the case of anomalous interaction linear in  $F^{\mu\nu}$  (i.e. for  $g_1 = 0$ )  $\varepsilon^2$  can be negative, provided  $n = p_3 = 0$ ,  $(\nu g - 1)eH < m^2$ . Thus the difficulty with complex energies indicated earlier for spin-1 equation [7] appears also for spin  $\frac{3}{2}$  equation [9]. However, this difficulty is overcome introducing anomalous interaction quadratic in  $F^{\mu\nu}$  (i.e. choosing  $g_1 \neq 0$  in (16)), namely

$$g_1 \leq -\frac{(3g - 2)^2 e}{72m^2}. \quad (23)$$

## 5 The charged particle with spin $\frac{3}{2}$ in electric and magnetic fields

In this case, as it was shown in [10, 11], we can confine our attention to the parallel and orthogonal configurations of electric and magnetic fields (all others configurations can be obtained ones mentioned in the above using Lorentz transformation)

**a)**  $\vec{E} \parallel \vec{H}$ . For constant, uniform  $\vec{E}$  and  $\vec{H}$  directed along  $z$  axis we may choose  $\vec{E} = (0, 0, E)$ ,  $\vec{H} = (0, 0, H)$ ,  $A_\mu = (x_3E, x_2H, 0, 0)$  and  $E \neq H$ . After substituting (19) into (16), equation (16) takes the form

$$\begin{aligned} & [(p_0 - ex_3E)^2 - (p_1 - ex_2H)^2 - p_2^2 - p_3^2 - m^2 \\ & + egS_3(H - iE) + 2eg_1S_3^2(H - iE)^2] \Phi_{\frac{3}{2}}^{(1)} = 0. \end{aligned} \quad (24)$$

Choosing  $\Phi_{\frac{3}{2}}^{(1)}$  in the form [10, 11]

$$\Phi_{\frac{3}{2}}^{(1)} = \exp(ip_1x_1 - \varepsilon x_0) f(x_2) \sum_{\nu=-\frac{3}{2}}^{\frac{3}{2}} g_\nu(x_3) \Omega_\nu^{\frac{3}{2}}, \quad (25)$$

where  $f(x_2)$  and  $g_\nu(x_3)$  are unknown functions, we can decompose equation (24) into two separate equations for  $f(x_2)$  and  $g_\nu(x_3)$ . After solving these equations,  $\Phi_{\frac{3}{2}}^{(1)}$  takes the form [10, 11]

$$\begin{aligned} \Phi_{\frac{3}{2}}^{(1)} &= \exp(ip_1x_2 - \varepsilon x_0) \exp\left(-\frac{(p_1 + ex_2H)^2}{2eH}\right) h_n(x_2) \\ &\times \exp\left(\frac{iz^2}{2}\right) \sum_{\nu=-\frac{3}{2}}^{\frac{3}{2}} G_\nu^j(-i\delta_\nu, -iz^2) \Omega_\nu^{\frac{3}{2}}, \quad j = 1, 2, \end{aligned} \quad (26)$$

where  $p_1, \varepsilon = \text{const}$ ,  $h_n(x_2) = \frac{H_n(x_2)}{\|H_n(x_2)\|}$ ,  $H_n(x_2)$  are Hermitian polynomials,  $z = \frac{1}{\sqrt{|eH|}}(\varepsilon - ex_2E)$ ,  $\delta_\nu = \frac{m^2 - \varepsilon\nu g(H - iE) - 2e\nu^2 g_1(H - iE)^2}{|eH|} - (2n + 1)$ ,  $n = 0, 1, 2, 3, \dots$ ,  $G_\nu^1(-i\delta_\nu, -iz^2) = F\left(\frac{1}{4}(1 - i\delta_\nu), \frac{1}{2}, -iz^2\right)$  and  $G_\nu^2(-i\delta_\nu, -iz^2) = F\left(\frac{1}{4}(1 - i\delta_\nu) + \frac{1}{2}, \frac{3}{2}, iz^2\right) \sqrt{iz^2}$ .  $F$  is the confluent hypergeometric function. So, in this case, the energy levels are not quantized.

**b)**  $\vec{E} \perp \vec{H}$ . Setting  $\vec{E} = (0, E, 0)$ ,  $\vec{H} = (0, 0, H)$ ,  $A_\mu = (x_2E, x_2H, 0, 0)$  we obtain following equation for  $\Phi_{\frac{3}{2}}^{(1)}$  instead of (24)

$$\begin{aligned} &[(p_0 - ex_2E)^2 - (p_1 - ex_2H)^2 - p_2^2 - p_3^2 - m^2 \\ &+ eg(S_3H - iS_2E) + 2eg_1(S_3H - iS_2E)^2] \Phi_{\frac{3}{2}}^{(1)} = 0. \end{aligned} \quad (27)$$

Representing  $\Phi_{\frac{3}{2}}^{(1)}$  in the form

$$\Phi_{\frac{3}{2}}^{(1)} = \exp(ip_1x_1 + ip_3x_3 - i\varepsilon x_0) \sum_{\nu=-\frac{3}{2}}^{\frac{3}{2}} P_\nu(x_2) \Omega_\nu^{\frac{3}{2}}, \quad (28)$$

substituting (28) into (5) and using transformation  $\hat{P}_\nu = U_{\nu\nu'} P_{\nu'}(x_2)$  we come to the equation

$$\left[ (\varepsilon - ex_2E)^2 - (p_1 - ex_2H)^2 + \frac{d^2}{dx_2^2} - p_3^2 - m^2 \right] \hat{P}_\nu(x_2) = eU_{\nu\nu'} \Lambda_{\nu'\nu''} U_{\nu''\nu'''}^{-1} \hat{P}_{\nu'''}(x_2). \quad (29)$$

where  $\Lambda_{\nu\nu'} = eg(S_3H - iS_2E)_{\nu\nu'} + 2eg_1(S_3H - iS_2E)_{\nu\nu'}^2$  and  $U_{\nu\nu'} \Lambda_{\nu'\nu''} U_{\nu''\nu'''}^{-1} = \lambda_\nu \delta_{\nu\nu'}$ .

The solution of equation (29) has the following form

$$E = H : \hat{P}_\nu(x_2) = \Phi(\alpha - 2eH\gamma(p_1 - \varepsilon)x_2), \quad (30)$$

where  $\Phi$  is Airy function,  $\alpha = (p_3^2 + p_1^2 + m^2 - \varepsilon^2) \gamma$  and  $\gamma = (4e^2h^2(p_1 - \varepsilon)^2)^{-\frac{1}{3}}$ . So, the energy levels are not quantized

$$E \neq H : \hat{P}_\nu(x_2) = \exp\left(\frac{iz^2}{2}\right) G_\nu^j(-ia_\nu, -iz^2), \quad j = 1, 2. \quad (31)$$

Here

$$a_\nu = -\left(\frac{p_3^2 + p_1^2 + m^2 - \varepsilon^2 + e\lambda_\nu}{e\eta} + \frac{(p_1H - \varepsilon E)^2}{e\eta^3}\right), \quad \eta = \sqrt{E^2 - H^2},$$

$$\lambda_\nu = -ig\nu\eta - 2g_1\nu^2\eta^2, \quad z = \sqrt{e\eta} \left( x_2 + \frac{p_1 H - \varepsilon E}{e\eta^2} \right).$$

When  $E > H$ ,  $iz^2$  of (31) becomes purely imaginary. So the energy levels are not quantized. In the case  $E < H$ ,  $iz^2$  becomes purely real and energy levels are quantized

$$\left( \varepsilon - \frac{p_1 E}{H} \right)^2 = \left( \frac{\eta'}{H} \right)^2 ((2n+1)\eta' + e\lambda_\nu + p_3^2 + m^2), \quad (32)$$

where  $\eta' = -i\eta$ ,  $n = 0, 1, 2, 3, \dots$ . If  $E \rightarrow 0$  we come to formula (22).

The exact form of matrix  $U_{\nu\nu'}$  which must diagonalize matrix  $\Lambda_{\nu\nu'}$  ( $U_{\nu\nu'}\Lambda_{\nu'\nu''}U_{\nu''\nu'''}^{-1} = \lambda_\nu\delta_{\nu\nu''}$ ,  $\lambda_\nu = -ig\nu\eta - 2g_1\nu^2\eta^2$ ) can be obtained from the equation

$$\sqrt{\left(\frac{3}{2} - \nu\right)\left(\frac{5}{2} + \nu\right)}U_{\nu\nu'+1} + 2(\nu - \lambda_\nu)U_{\nu\nu'} + \sqrt{\left(\frac{3}{2} + \nu\right)\left(\frac{5}{2} - \nu\right)}U_{\nu\nu'-1} = 0,$$

$$\nu = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \quad U_{\nu, \frac{5}{2}} = U_{\nu, -\frac{5}{2}} = 0.$$

## 6 Discussion

Thus we present the equation for particles with spin  $\frac{3}{2}$  interacting with electromagnetic field, which is casual (it can be proved using approach proposed in [7]), i.e. their solutions are propagated with the velocity smaller than the light velocity. We also find the solutions of this equation in constant magnetic field and in static electric field inclined at an arbitrary angle to a static magnetic field. As it was shown above the corresponding constant  $g_1$  can be chosen in such form in which the energies levels of charged particle with spin  $\frac{3}{2}$  in constant magnetic field are not complex for arbitrary values  $H$  and  $g$ .

## Acknowledgements

I would like to thank Prof. A. Nikitin for useful discussion and suggestions.

- [1] Velo G. and Zwanziger D., Propagation and quantization of Rarita–Schwinger waves in an external electromagnetic potential, *Phys. Rev. D*, 1969, V.186, N 5, 218–226.
- [2] Fierz M. and Pauli W., On relativistic wave equations for particles of arbitrary spin in an electromagnetic field, *Proc. R. Soc. A*, 1939, V.173, 211–232.
- [3] Hurley W.J., Relativistic wave equations for particles with arbitrary spin, *Phys. Rev. D*, 1971, V.4, N 9, 3605–3616.
- [4] Wightman A.S., Invariant wave equations: general theory and applications to the external field problem, Invariant wave equation, Lecture Notes in Physics, Berlin, Springer, 1978, V.73, 1–101.
- [5] Lomont J.S. and Moses H.E., Dirac-like wave equations for particles of nonzero rest mass and their quantization, *Phys. Rev.*, 1960, V.118, N 1, 337–344.
- [6] Loide R.K., Kõiv M. and Saar R., Single mass equations for an antisymmetric tensor-bispinor, *J. Phys. A*, 1983, V.16, N 1, 463–467.
- [7] Fushchich W.I. and Nikitin A.G., Symmetry of equation of quantum mechanics, New York, Allerton, 1994.
- [8] Niederle J. and Nikitin A.G., Relativistic wave equations for interacting massive particles with arbitrary half-integer spin, *Phys. Rev. D*, 2001, V.64, N 12, 125013.
- [9] Galkin A., Tensor-bispinor equation for doublets, in Proceedings of the Institute of Mathematics of NAS of Ukraine “Group and Analytic Methods in Mathematical Physics”, 2001, V.36, 67–77.
- [10] Lam L., Motion in electric and magnetic fields. I. Klien–Gordon particles, *J. Math. Phys.*, 1971, V.12, N 2, 299–303.
- [11] Susloparov V.M., Relativistic particle with arbitrary spin in electric and magnetic fields, *Ukr. Phys. J.*, 1984, V.29, N 8, 1265–1267.