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Exact Solutions of Matrix Generalizations of Some Integrable Systems

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Exact solutions of a matrix generalization of nonlinear Yajima–Oikawa model are built in an explicit form. The Melnikov-like system was also integrated.

1 Introduction

The hierarchy of Kadomtsev–Petviashvili equations can be given as an infinite sequence of the Sato–Wilson operator equations [1, 2]

$$\alpha_n W_{t_n} = -\left(W\mathcal{D}^n W^{-1}\right)_- W, \qquad n\in\mathbb{N}, \quad \alpha_n\in\mathbb{C}, \tag{1}$$

where $W = 1 + w_1\mathcal{D}^{-1} + w_2\mathcal{D}^{-2} + \cdots$ is a microdifferential operator (MDO) with coefficients w_i, $i\in\mathbb{N}$, depending on the variables $\boldsymbol{t} = (t_1, t_2, \ldots)$, $t_1 := x$ and $\mathcal{D} := \frac{\partial}{\partial x}$, $\mathcal{D}\mathcal{D}^{-1} = 1$. Differential and integral parts of the microdifferential operator $W\mathcal{D}^n W^{-1}$ are denoted by $\left(W\mathcal{D}^n W^{-1}\right)_+$ and $\left(W\mathcal{D}^n W^{-1}\right)_-$ respectively. In the algebra MDO ζ:

$$\zeta = \left\{ \sum_{i=-\infty}^{n(L)} a_i\mathcal{D}^i : a_i = a_i(\boldsymbol{t}) \in \mathcal{A};\ i, n(L)\in\mathbb{Z} \right\},$$

the operation of multiplication is induced by the generalized Leibnitz rule

$$\mathcal{D}^n f := \sum_{j=0}^{\infty} \binom{n}{j} f^{(j)}\mathcal{D}^{n-j}, \quad n\in\mathbb{Z}, \qquad \mathcal{D}^m(f) := \frac{\partial^m f}{\partial x^m} = f^{(m)}, \quad m\in\mathbb{Z}_+,$$

where $\mathcal{D}^n\mathcal{D}^m := \mathcal{D}^m\mathcal{D}^n := \mathcal{D}^{n+m}$, $n, m\in\mathbb{Z}$, and f is the operator of multiplication by a function $f(\boldsymbol{t})$, which belongs to the same functional space $\mathcal{A}$ that the coefficients of microdifferential operators $L\in\zeta$.

With the aid of the MDO L is defined by formula $L := W\mathcal{D}W^{-1} = \mathcal{D} + U\mathcal{D}^{-1} + U_2\mathcal{D}^{-2} + \cdots$ system (1) can be rewritten in the form of the Lax representation

$$\alpha_n L_{t_n} = [B_n, L] := B_n L - L B_n, \tag{2}$$

where $B_n = (L^n)_+ = (W\mathcal{D}^n W^{-1})_+$, $n\in\mathbb{N}$.

Nonlocal reduced hierarchy of Kadomtsev–Petviashvili is the system of operator equations (2) with the additional restriction so-called *k-constraint* of the form [3, 4, 5, 6, 7] (see also [8]) $L^k := (L^r)^k = B_k + \sum\limits_{i=1}^{l} q_i\mathcal{D}^{-1}r_i^\top$, where "$\top$" denotes transposition which is in accordance with dynamics of system (2), if field-variables q_i, r_i satisfy the system of the following equations:

$$\alpha_n q_{it_n} = B_n(q_i), \qquad \alpha_n r_{it_n} = -B_n^\tau(r_i),$$

the symbol "τ" denotes the transposition of differential operator.

Equations from k–reduced hierarchy of Kadomtsev–Petviashvili allow the Lax representation

$$\left[B_k + \boldsymbol{q}\mathcal{D}^{-1}\boldsymbol{r}^\top, \alpha_n\partial_{t_n} - B_n\right] = 0, \qquad n \in \mathbb{N}. \tag{3}$$

2 Exact solutions of a matrix generalization of Yajima–Oikawa model

In the present paper we consider the matrix case of (3): $k = 2$, $n = 2$ and $U_1 := U, U_2, U_3, \ldots \in \mathrm{Mat}_{N\times N}(\mathbb{C})$, $\boldsymbol{q}, \boldsymbol{r} \in \mathrm{Mat}_{N\times N'}(\mathbb{C})$ and obtain the system:

$$\alpha_2\boldsymbol{q}_{t_2} = \boldsymbol{q}_{xx} + 2U\boldsymbol{q}, \qquad \alpha_2 U_{t_2} = (\boldsymbol{q}\mathbf{r}^\top)_x, \qquad \alpha_2\boldsymbol{r}_{t_2} = -\boldsymbol{r}_{xx} - 2\boldsymbol{r}U. \tag{4}$$

Introduce the additional reductions of complex conjugation $\alpha_2 = i$, $t_2 = t$, $U = U^* := \bar{U}^\top$, $\boldsymbol{r} = i\bar{\boldsymbol{q}}M^\top$, where $M \in \mathrm{Mat}_{N'\times N'}(\mathbb{C}), M = M^*$. System (4) can be represented as:

$$i\boldsymbol{q}_t = \boldsymbol{q}_{xx} + 2U\boldsymbol{q}, \qquad U_t = (\boldsymbol{q}M\boldsymbol{q}^*)_x. \tag{5}$$

System (5) is a matrix generalization of Yajima–Oikawa model [9]. Operators of this system in the Lax representation ($[L, A] = 0$) have the form:

$$L = \mathcal{D}^2 + 2U + i\boldsymbol{q}M\mathcal{D}^{-1}\boldsymbol{q}^*, \qquad A = i\partial_t - \mathcal{D}^2 - 2U.$$

Proposition 1 ([2, 10]). *Let $B = B_+$ be a differential operator; $\boldsymbol{f}\mathcal{D}^{-1}\boldsymbol{g}$, $\tilde{\boldsymbol{f}}\mathcal{D}^{-1}\tilde{\boldsymbol{g}} \in \zeta$. Then the following relations hold:*

$$B\boldsymbol{f}\mathcal{D}^{-1}\boldsymbol{g}^\top = \left(B\boldsymbol{f}\mathcal{D}^{-1}\boldsymbol{g}^\top\right)_+ + B(\boldsymbol{f})\mathcal{D}^{-1}\boldsymbol{g}^\top,$$
$$\boldsymbol{f}\mathcal{D}^{-1}\boldsymbol{g}^\top B = \left(\boldsymbol{f}\mathcal{D}^{-1}\boldsymbol{g}^\top B\right)_+ + \boldsymbol{f}\mathcal{D}^{-1}\left(B^\tau\boldsymbol{g}\right)^\top,$$
$$\boldsymbol{f}\mathcal{D}^{-1}\boldsymbol{g}^\top\tilde{\boldsymbol{f}}\mathcal{D}^{-1}\tilde{\boldsymbol{g}}^\top = \boldsymbol{f}\left(\int \boldsymbol{g}^\top\tilde{\boldsymbol{f}}\right)\mathcal{D}^{-1}\tilde{\boldsymbol{g}}^\top - \boldsymbol{f}\mathcal{D}^{-1}\left(\int \boldsymbol{g}^\top\tilde{\boldsymbol{f}}\right)\tilde{\boldsymbol{g}}^\top. \tag{6}$$

In formulas (6) the symbol $\int \boldsymbol{g}^\top\tilde{\boldsymbol{f}}$ stands for an arbitrary fixed primitive of $\left(\boldsymbol{g}^\top\tilde{\boldsymbol{f}}\right)(x, t_2)$ as a function of x.

Let φ, ψ be smooth complex matrix $(N \times K)$ functions of real variables $x, t_2 \in \mathbb{R}$, $C = (C_{mn}) = \mathrm{const} \in \mathrm{Mat}_{K\times K}(\mathbb{C})$, and also:

1) the improper integral $\int_{-\infty}^x \psi^\top\varphi\, ds := \int_{-\infty}^x \psi^\top(s, t_2)\varphi(s, t_2)\, ds$ converges absolutely $\forall\ (x, t_2) \in \mathbb{R}\times\mathbb{R}_+$ and admits differentiation by the parameter $t_2 \in \mathbb{R}_+$;

2) the matrix-function $\Omega(x, t_2) := C + \int_{-\infty}^x \psi^\top\varphi\, ds$ is nondegenerate in $(x, t_2) \in \sigma \subset \mathbb{R}\times\mathbb{R}_+$.

Define the functions $\Phi = \Phi(x, t_2)$, $\Psi = \Psi(x, t_2)$ and MDO W by the following way:

$$\Phi = \varphi\Omega^{-1}, \qquad \Psi^\top = \Omega^{-1}\psi^\top, \qquad W = 1 - \Phi\mathcal{D}^{-1}\psi^\top. \tag{7}$$

Lemma 1. *The components Φ_{ij}, Ψ_{ij}, $i = \overline{1, N}$, $j = \overline{1, K}$, of matrix functions Φ, Ψ (7) can be given as:*

$$\Phi_{ij} = \left(\varphi\Omega^{-1}\right)_{ij} = (-1)^{K+j}\frac{\begin{vmatrix}\Omega_{(j)}\\ \varphi_i\end{vmatrix}}{|\Omega|}, \tag{8}$$

$$\Psi_{ij} = \left(\psi\Omega^{\top -1}\right)_{ij} = (-1)^{K+j}\frac{\begin{vmatrix}\Omega^\top_{(j)}\\ \psi_i\end{vmatrix}}{|\Omega|}. \tag{9}$$

Here $\Omega_{(j)}$ is obtained from Ω by deletion of j-line; φ_i, ψ_i are i-lines of matrixes φ, ψ.

Proof. In order to prove (8), (9) we use a well-known algebraic equality for framed determinant:

$$\det\begin{pmatrix}\Omega & \psi_j^\top\\ \varphi_i & \alpha\end{pmatrix} := \begin{vmatrix}\Omega & \psi_j^\top\\ \varphi_i & \alpha\end{vmatrix} = \alpha\det\Omega - \varphi_i\Omega^C\psi_j^\top,$$

where Ω^C is the matrix of cofactors.

$$\Phi_{ij} = \left(\varphi\Omega^{-1}\right)_{ij} = \varphi_i\Omega^{-1}e_j^\top = (-1)^{K+j}\frac{\begin{vmatrix}\Omega_{(j)}\\ \varphi_i\end{vmatrix}}{|\Omega|}.$$

Here $e_i = (e_{i_1}, \dots, e_{i_K})$, $e_{i_i} = 1$, $e_{i_j} = 0$ for $i, j = \overline{1, K}$, $i \neq j$.

By the similar reasoning, formula (9) can be proved. ■

Theorem 1 ([10]). *MDO W has an inverse operator W^{-1} and:*

$$W^{-1} = 1 + \varphi\mathcal{D}^{-1}\Psi^\top.$$

Proposition 2. *For MDO W (7) the equalities are true:*

$$\begin{aligned}
W\mathcal{D}^2W^{-1} &= \left(I - \Phi\mathcal{D}^{-1}\psi^\top\right)\mathcal{D}^2\left(I + \varphi\mathcal{D}^{-1}\Psi^\top\right) = \mathcal{D}^2 + 2\left(\varphi\Omega^{-1}\psi^\top\right)_x\\
&- \Phi\mathcal{D}^{-1}\left(\psi_{xx}^\top - \int_{-\infty}^x \psi_{ss}^\top\varphi\, ds\,\Psi^\top\right) + \left(\varphi_{xx} - \Phi\int_{-\infty}^x \psi^\top\varphi_{ss}\, ds\right)\mathcal{D}^{-1}\Psi^\top,\\
W\left(i\partial_t - \mathcal{D}^2\right)W^{-1} &= i\partial_t - \mathcal{D}^2 - 2\left(\varphi\Omega^{-1}\psi^\top\right)_x\\
&+ \Phi\mathcal{D}^{-1}\left\{\left(i\psi_t^\top + \psi_{xx}^\top\right) - \int_{-\infty}^x\left(i\psi_t^\top + \psi_{ss}^\top\right)\varphi\, ds\,\Psi^\top\right\}\\
&+ \left\{(i\varphi_t - \varphi_{xx}) - \Phi\int_{-\infty}^x \psi^\top(i\varphi_t - \varphi_{ss})\, ds\right\}\mathcal{D}^{-1}\Psi^\top.
\end{aligned}$$

The proof of the Proposition 2 is based on the using of formulas (6) and the generalized Leibnitz rule.

Consider operators $L_0 = \mathcal{D}^2$, $A_0 = i\partial_t - \mathcal{D}^2$, $\hat{L} = WL_0W^{-1}$, $\hat{A} = WA_0W^{-1}$.

Theorem 2. *Let:*

a) φ be a solution of the equation $i\varphi_t = \varphi_{xx}$;

b) $\varphi_{xx} = \varphi\Lambda$, where $\Lambda = \operatorname{diag}\left(\lambda_1^2, \lambda_2^2, \dots, \lambda_K^2\right) = \mathrm{const} \in \mathrm{Mat}_{K\times K}(\mathbb{C})$;

c) $\psi = \bar{\varphi}$;

d) $C = C^*$.

Then

1) $\Psi = \bar{\Phi}$;

2) $\hat{L} = \mathcal{D}^2 + 2\left(\varphi\Omega^{-1}\varphi^*\right)_x + \Phi J\mathcal{D}^{-1}\bar{\Phi}^\top$, where $J = C\Lambda - \Lambda^*C$;

3) $\hat{A} = i\partial_t - \mathcal{D}^2 - 2\left(\varphi\Omega^{-1}\varphi^*\right)_x$.

Proof. 1) From definitions (7) and condition d we have:

$$\bar{\Phi} = \overline{\varphi\left(C + \int_{-\infty}^x \varphi^*\varphi\, ds\right)^{-1}} = \bar{\varphi}\left(\bar{C} + \int_{-\infty}^x \varphi^\top\bar{\varphi}\, ds\right)^{-1} = \psi\left(C^\top + \int_{-\infty}^x \varphi^\top\psi\, ds\right)^{-1} = \Psi.$$

2) From Proposition 2, condition b and properties:

$$\Phi\int_{-\infty}^x \psi^\top\varphi\, ds = \varphi - \Phi C \qquad \text{and} \qquad \int_{-\infty}^x \psi^\top\varphi\, ds\Psi^\top = \psi^\top - C\Psi^\top,$$

it follows that:

$$\hat{L} = \mathcal{D}^2 + 2\left(\varphi\Omega^{-1}\varphi^*\right)_x - \Phi\mathcal{D}^{-1}\Lambda^*\psi^\top + \Phi\mathcal{D}^{-1}\Lambda^*\int_{-\infty}^x \psi^\top\varphi\, ds\Psi^\top$$
$$+\varphi\Lambda\mathcal{D}^{-1}\Psi^\top - \Phi\int_{-\infty}^x \psi^\top\varphi\, ds\Lambda\mathcal{D}^{-1}\Psi^\top = \mathcal{D}^2 + 2\left(\varphi\Omega^{-1}\varphi^*\right)_x + \Phi\left(C\Lambda - \Lambda^*C\right)\mathcal{D}^{-1}\Psi^\top.$$

3) The validity of this item follows from Proposition 2 and conditions a), c). ■

Proposition 3. *The matrix $J = (J_{mn})$, $m, n = \overline{1, K}$ has the following properties:*
1) $J = -J^*$;
2) $J_{mn} = C_{mn}\left(\lambda_m^2 - \overline{\lambda_n^2}\right)$;
3) if the matrix C is diagonal, then: $J = \operatorname{diag}\left(2ic_1 \operatorname{Im}\lambda_1^2, 2ic_2 \operatorname{Im}\lambda_2^2, \ldots, 2ic_K \operatorname{Im}\lambda_K^2\right)$.

Proof. 1) $J^* = (C\Lambda - \Lambda^*C)^* = \Lambda^*C^* - C^*\Lambda = \Lambda^*C - C\Lambda = -J$.
The proof of the 2), 3) is based on the using of formulas of operations with matrices. ■

Corollary 1. *Let the matrix J be defined by the following condition: $\Phi J\Phi^* = i\boldsymbol{q}M\boldsymbol{q}^*$, then the functions $\boldsymbol{q} = (q_{ij})$ and $U = (u_{kl})$, $i, k, l = \overline{1, N}$, $j = \overline{1, K}$, where*

$$q_{ij} = (-1)^{j+K}\frac{\begin{vmatrix}\Omega_{(j)}\\ \varphi_i\end{vmatrix}}{|\Omega|}, \qquad u_{kl} = \left(\begin{vmatrix}\Omega & \bar{\varphi}_l^\top\\ \varphi_k & 0\end{vmatrix}|\Omega|^{-1}\right)_x$$

are solutions of system (5).

The proof of corollary is based on the using of formula (8), equality for framed determinant and Theorem 2.

Consider the simplest case of matrix equation (5): $N = 2$, $K = 1$. Then $\varphi_1 = \hat{c}e^{\lambda x - i\lambda^2 t}$, $\varphi_2 = \hat{c}e^{-\lambda x - i\lambda^2 t}$, $M = \mu \in \mathbb{R}$ and under conditions $\operatorname{Re}\lambda > 0$, $\mu = 4\operatorname{Re}\lambda\operatorname{Im}\lambda \cdot C$ solutions have the form:

$$q_1 := q_{11} = \frac{4\hat{c}\operatorname{Re}\lambda\operatorname{Im}\lambda\, e^{\lambda x - i\lambda^2 t}}{\mu + 4|\hat{c}|^2\operatorname{Im}\lambda\,\sinh\left(2\operatorname{Re}\lambda\cdot x\right)e^{4\operatorname{Re}\lambda\operatorname{Im}\lambda\, t}},$$
$$q_2 := q_{21} = \frac{4\hat{c}\operatorname{Re}\lambda\operatorname{Im}\lambda\, e^{-\lambda x - i\lambda^2 t}}{\mu + 4|\hat{c}|^2\operatorname{Im}\lambda\,\sinh(2\operatorname{Re}\lambda\cdot x)e^{4\operatorname{Re}\lambda\operatorname{Im}\lambda\, t}},$$
$$u_{11} = \frac{4|\hat{c}|^2\operatorname{Re}\lambda\operatorname{Im}\lambda\, e^{2\operatorname{Re}\lambda\, x + 4\operatorname{Re}\lambda\operatorname{Im}\lambda\, t}}{\mu + 4|\hat{c}|^2\operatorname{Im}\lambda\,\sinh(2\operatorname{Re}\lambda\cdot x)e^{4\operatorname{Re}\lambda\operatorname{Im}\lambda\, t}},$$
$$u_{12} = \bar{u}_{21} = \frac{4|\hat{c}|^2\operatorname{Re}\lambda\operatorname{Im}\lambda\, e^{4\operatorname{Re}\lambda\operatorname{Im}\lambda\, t + 2i\operatorname{Re}\lambda\, x}}{\mu + 4|\hat{c}|^2\operatorname{Im}\lambda\,\sinh(2\operatorname{Re}\lambda\cdot x)e^{4\operatorname{Re}\lambda\operatorname{Im}\lambda\, t}},$$
$$u_{22} = \frac{4|\hat{c}|^2\operatorname{Re}\lambda\operatorname{Im}\lambda\, e^{-2\operatorname{Re}\lambda x + 4\operatorname{Re}\lambda\operatorname{Im}\lambda\, t}}{\mu + 4|\hat{c}|^2\operatorname{Im}\lambda\,\sinh(2\operatorname{Re}\lambda\cdot x)e^{4\operatorname{Re}\lambda\operatorname{Im}\lambda\, t}}.$$

3 Exact solutions of higher equation from matrix hierarchy of Yajima–Oikawa

Let us consider the following operators:

$$L = \mathcal{D}^2 + 2U + i\boldsymbol{q}M\mathcal{D}^{-1}\boldsymbol{q}^*, \qquad A = \partial_t - \mathcal{D}^3 - 3U\mathcal{D} - \frac{3}{2}U_x - \frac{3}{2}i\boldsymbol{q}M\boldsymbol{q}^*.$$

The result of equation $[L, A] = 0$ will be the system:

$$\boldsymbol{q}_t = \boldsymbol{q}_{xxx} + 3U\boldsymbol{q}_x + \frac{3}{2}U_x\boldsymbol{q} + \frac{3}{2}i\mu\boldsymbol{q}\boldsymbol{q}^*\boldsymbol{q}, \tag{10}$$

$$U_t = \frac{1}{4}U_{xxx} + 3UU_x + \frac{3}{4}i\mu(\boldsymbol{q}_{xx}\boldsymbol{q}^* - \boldsymbol{q}\boldsymbol{q}^*_{xx}). \tag{11}$$

This system is a matrix generalization of Melnikov model [7, 11].

Proposition 4. *For MDO W the equality is true:*

$$\begin{aligned} W\left(\partial_t - \mathcal{D}^3\right)W^{-1} &= \partial_t - \mathcal{D}^3 - 3\left(\varphi\Omega^{-1}\psi^\top\right)_x\mathcal{D} \\ &- \frac{3}{2}\left(\varphi_{xx}\Omega^{-1}\psi^\top - \varphi\Omega^{-1}\psi^\top_{xx} + \varphi\Omega^{-1}\psi^\top_x - \varphi\Omega^{-1}\psi^\top\varphi_x\Omega^{-1}\psi^\top\right) \\ &+ \Phi\mathcal{D}^{-1}\left\{\left(\psi^\top_t - \psi^\top_{xxx}\right) - \int_{-\infty}^x \left(\psi^\top_t - \psi^\top_{sss}\right)\varphi\, ds\, \Psi^\top\right\} \\ &+ \left\{(\varphi_t - \varphi_{xxx}) - \Phi\int_{-\infty}^x \psi^\top(\varphi_t - \varphi_{sss})\, ds\right\}\mathcal{D}^{-1}\Psi^\top. \end{aligned}$$

The proof of the proposition is based on the formulas (6).

Consider operators $L_0 = \mathcal{D}^2$, $A_0 = \partial_t - \mathcal{D}^3$, $\hat{L} = WL_0W^{-1}$, $\hat{A} = WA_0W^{-1}$.

Theorem 3. *Let:*

a) φ be a solution of the equation $\varphi_t = \varphi_{xxx}$;

b) $\varphi_{xx} = \varphi\Lambda$, where $\Lambda = \operatorname{diag}\left(\lambda_1^2, \lambda_2^2, \ldots, \lambda_K^2\right) = \mathrm{const} \in \mathrm{Mat}_{K\times K}(\mathbb{C})$;

c) $\psi = \bar{\varphi}$;

d) $C = C^*$.

Then

1) $\hat{L} = \mathcal{D}^2 + 2\left(\varphi\Omega^{-1}\varphi^*\right)_x + \Phi J\mathcal{D}^{-1}\bar{\Phi}^\top$, where $J = C\Lambda - \Lambda^*C$;

2) $\hat{A} = \partial_t - \mathcal{D}^3 - 3\left(\varphi\Omega^{-1}\varphi^*\right)_x\mathcal{D} - \frac{3}{2}\left(\varphi_{xx}\Omega^{-1}\varphi^* - \varphi\Omega^{-1}\varphi^*_{xx} + \varphi\Omega^{-1}\varphi^*_x - \varphi\Omega^{-1}\varphi^*\varphi_x\Omega^{-1}\varphi^*\right)$.

Proof. 2) The validity of this item follows from Proposition 4 and conditions a), c). ■

Remark 1. For system (10) the corollary of the previous part is true (see above).

Consider the case $N = 2$, $K = 1$. Then $\varphi_1 = \hat{c}e^{\lambda x + \lambda^3 t}$, $\varphi_2 = \hat{c}e^{-\lambda x - \lambda^3 t}$, $M = \mu \in \mathbb{R}$ and under conditions $\operatorname{Re}\lambda > 0$, $\mu = 4\operatorname{Re}\lambda\operatorname{Im}\lambda\cdot C$ solutions will be of the form:

$$q_1 := q_{11} = \frac{4\hat{c}\operatorname{Re}\lambda\operatorname{Im}\lambda\, e^{\lambda x + \lambda^3 t}}{\mu + 4|\hat{c}|^2\operatorname{Im}\lambda\cdot\sinh\left(2\operatorname{Re}\lambda\cdot x + 2\left(\operatorname{Re}^3\lambda - 3\operatorname{Re}\lambda\operatorname{Im}^2\lambda\right)t\right)},$$

$$q_2 := q_{21} = \frac{4\hat{c}\operatorname{Re}\lambda\operatorname{Im}\lambda\, e^{-\lambda x - \lambda^3 t}}{\mu + 4|\hat{c}|^2\operatorname{Im}\lambda\cdot\sinh\left(2\operatorname{Re}\lambda\cdot x + 2\left(\operatorname{Re}^3\lambda - 3\operatorname{Re}\lambda\operatorname{Im}^2\lambda\right)t\right)},$$

$$u_{11} = \frac{4|\hat{c}|^2\operatorname{Re}\lambda\operatorname{Im}\lambda\, e^{2\left(\operatorname{Re}\lambda\cdot x + \left(\operatorname{Re}^3\lambda - 3\operatorname{Re}\lambda\operatorname{Im}^2\lambda\right)t\right)}}{\mu + 4|\hat{c}|^2\operatorname{Im}\lambda\cdot\sinh\left(2\operatorname{Re}\lambda\cdot x + 2\left(\operatorname{Re}^3\lambda\cdot x - 3\operatorname{Re}\lambda\operatorname{Im}^2\lambda\right)t\right)},$$

$$u_{12} = \bar{u}_{21} = \frac{4|\hat{c}|^2\operatorname{Re}\lambda\operatorname{Im}\lambda\, e^{2i\left(\operatorname{Im}\lambda\cdot x + \left(3\operatorname{Re}^2\lambda\operatorname{Im}\lambda - \operatorname{Im}^3\lambda\right)t\right)}}{\mu + 4|\hat{c}|^2\operatorname{Im}\lambda\cdot\sinh\left(2\operatorname{Re}\lambda\cdot x + 2\left(\operatorname{Re}^3\lambda\cdot x - 3\operatorname{Re}\lambda\operatorname{Im}^2\lambda\right)t\right)},$$

$$u_{22} = \frac{4|\hat{c}|^2\operatorname{Re}\lambda\operatorname{Im}\lambda\, e^{-2\left(\operatorname{Re}\lambda\cdot x + \left(\operatorname{Re}^3\lambda - 3\operatorname{Re}\lambda\operatorname{Im}^2\lambda\right)t\right)}}{\mu + 4|\hat{c}|^2\operatorname{Im}\lambda\cdot\sinh\left(2\operatorname{Re}\lambda\cdot x + 2\left(\operatorname{Re}^3\lambda\cdot x - 3\operatorname{Re}\lambda\operatorname{Im}^2\lambda\right)t\right)}.$$

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