Proceedings of Institute of Mathematics of NAS of Ukraine 2002, Vol. 43, Part 2, 589–601

New Relationships and Measurements for Gravity Physics

Orest BEDRIJ

Institute of Mathematical Physics, P.O. Box 97, Monmouth Beach, NJ 07750, USA

The suggested formulation of the laws governing the physics of gravitation provides new phenomenological considerations for a *mathematical method of elucidating and measuring phenomena.* A systematic treatment with broader conceptual framework, than the conventional formalism is presented advancing new physical relationships and fundamental constants that are based on known fundamental constants, physical relationships and high precision measurements.

1 Introduction

We discuss foundational principles of characterizing and formulating the laws governing the physics of gravity. The suggested formulation of the laws governing the physics of gravitation provides new phenomenological considerations/correlations for a *mathematical method of elucidating and measuring phenomena.* A systematic treatment with broader conceptual framework, than the conventional formalism is presented advancing new physical relationships and fundamental constants that are based on known fundamental constants, physical relationships and high precision measurements. Theory (symmetries, scale-invariance, singularities, the Principle of the One-and-the Many) [1], measurements (fundamental constants in quantum electrodynamics [2], and nucleon-meson dynamics [3]) are united. Simple algebra of logical/measurable evidence to predict: 1) physical relationships/quantities, 2) fundamental physical constants, and 3) the basic units of quantities, for the laws governing the physics of gravity are utilized. The suggested representation permits mathematical characterization/testing of new phenomena based on measurements [2], enabling one to calculate/determine new physical relationships [1, 3] and the nature of unmeasured reality. *New quantities* (**) and *relationships* (*) for the: (gravitational flux density, penetrability, potential density, field quantum, resonance condition, gravitance)** and (gravitational field strength, Newton's gravitational constant, mass [flux], gravitational potential, gravitational force)* are suggested. For comparative purposes, the Earth's dimensions and the values of the electron, proton, and neutron constants, as they relate to the fundamental equations of gravitation are given.

Symmetry is a very useful tool in the group theoretical physics [4]. It has been suggested by some authors (Lie [5], Lorenz [6], Einstein [7], Poincaré [8], Heaviside [9], Bateman [10], Cunningham [11], Rainich [12]), that symmetries of Maxwell and Dirac equations, as well as, supersymmetry (a symmetry that connects elementary particles of integer/half-integer spin in common symmetry multiplets, Weinberg [13]), and other differential equations of quantum mechanics [14], produce immensely valuable fundamental results. We suggest: in addition to [4–14] approaches, symmetries (in particular group symmetries) be integrated with fundamental constants and the laws of physics in scale-invariant relationships [1] (see Tables 2, 3, 4), resulting in a very effective phenomenological means to: 1) discover new phenomena; 2) formulate, verify, and elucidate the foundation of physics and astrophysics in general [15], and in particular the broad range nature of gravitation [1, 2, 3]. The suggested formulation places a restriction on the possible solutions of the laws governing the physics of 'gravity', permitting general relationships than those allowable by usual interpretation. Facilitating system of equations within the formalism of wave mechanics [16, 17], an "observer", for continuous characterization and measurement of phenomena.

The validity of any fundamental equation rests in its agreement with experiment. The severe constraint of invariance, normalization and scale changes [3, 15], and the symmetry principle, enables one to advance toward the *invariable foundation of physics* (the Highest Common Factor: nature of invariable/unmeasured reality, where, as we have seen [1, 2, 3], the standard formalism is incomplete) and compute equations from a wide range of probabilities. There is only one system of Poincaré-invariant partial differential equations of first order, for two real vectors $\boldsymbol{E}$ and $\boldsymbol{H}$. This is the system, which translates to Maxwell's equations [4]. It is feasible to "derive" the Dirac, Schrödinger, electromagnetic field [14], and other equations [2, 16, 17] in a comparable manner. It is this rigorous constraint that causes energy quantization. Correspondingly, the equations of Newton, Maxwell, Poincaré, Laplace, d'Alembert, Euler–Lagrange, Lame, and Hamilton–Jacobi have a very high symmetry [4]. It is this high symmetry which is the property distinguishing these equations from other ones considered by physicists and mathematicians.

2 General principles

Central to our methodology is the *singularity* '**1**' (the Principle of the Initial Conditions of measurement: the *dimensionless point*, discussed in [1, 2, 3, 18, 19]. Just as each number, on the mathematical scale, has a unifying principle (zero) as its' *starting frame of reference*, so each physical quantity (and the laws of physics), on the *physics scale of quantities* ([1] equation (2), or [18] equation (4)), has a unifying principle as its' starting frame of reference: the initial conditions of measurement, as in $\mathbf{1} = E/mc^2$. The '**1**' serves also as the experimental underpinning, normalization condition, and the scale-invariant equilibrium frame of reference for E and mc^2 [1]. In this same vein, E serves as surrogate (proxy) equilibrium, and scale-invariant frame of reference for m and c^2 [18]. The *surrogate equilibrium* frame of reference (singularity) defines the *Principle of the Final Condition* of measurement, with an equality ('='), as in $E = mc^2$.

The '**1**' and the equality '=' are *dimensionless points* (law of physics singularities $\gamma = 0$) with vast power to describe the nature of invariable/unmeasured reality. The '**1**' and the '=' represent the *a priori principle of physics* (invariance), and a natural location for the "collapse of the wave function", the *points of inversion* and measurement, also called the "quantum jump" or the point of amplification, which manifests a sharp increase in output signal when (via variation of the magnetic field) the Zeeman splitting frequency is varied through the cavity resonant frequency. It is the '**1**' and the '=' that place a restriction on the possible solutions of the Schrödinger equation: a restriction [3] that leads to energy quantization. In the logarithmic, or the natural log scale, the equilibrium frame of reference (invariance) '**1**' $\to 0$, i.e. $\gamma = 0$ ($\mathbf{1} = 10^0 = e^0 = x^0$) [1].

The concept of the *zero* ('0'), goes back around 300 BC to Babylonians, who used two slanted wedges to represent an *empty space*. In mathematics the concept of the zero has been developed. However, in the formalism of physics, the notion of the singularity (*a priori*) has not been adequately defined/developed with mathematical/experimental formalism. Namely, what its positional notation (the Principle of Position [18]) or fundamental nature is, how it behaves with physical quantities/fundamental constants, or how it may be generalized. The quantum interpretation is not characterizing the *nature of singularity*, but the relationship between reality and its representation, the proxy wave ψ.

Mathematics is our universal language. When *validated by experiment*, mathematics becomes our generalized language of the laws of physics. We know how zero interrelates with numbers, and those numbers with one another. These descriptions take form of the laws governing their interactions. The effect of such laws brings zero and numbers closer together. It changes our understanding of numbers themselves. If you look at a *singularity* $\gamma = 0$ you see a single *dimensionless point*; but glimpse through the singularity and you will see the universe [1, equations (2)–(12)]. At $\gamma = 0$ the concept of space-time loses its meaning. Einstein's equations are violated (i.e., collapse of the wave function: the essence of Gödel's Incompleteness Theorem of 1930), with reality becoming *indeterminable* to the observer, and that human beings will ever expose all ultimate secrets of the universe. For zero (singularity) to be the possibility of universal significance with what it gives power to, we must understand how to add/subtract with it, for a start, replacing it with variety of words for the same thing with concise rules for zero/numbers.

In the Copenhagen interpretation, all the unexplained transitions among the classical/quantum physics occur at the *boundary* connecting measuring/quantum system. We suggest that physical quantities, atoms, and galaxies are the 'quantum entities' and 'observers' (John Wheeler's participators): Georg Cantor's sets, with their own structure and physical laws, that have order (the Principle of Order [18]), endless hierarchy of infinities and *sequence* (the Principle of Position [18]). As a final point, the whole universe may be drawn in as observers: participators: sets: physical quantities, while the boundary between measuring and quantum system is the '**1**' and the '=' points in the unmeasured reality.

Einstein (1924), Dirac (1937), Teller (1948), Landau (1955), Brans and Dicke (1961), DeWitt (1964), Isham, Salam and Stratdhdee (1971), Salam and Wigner (1972), and others have suggested a variety of approaches leading to a relation between gravitation, electromagnetism, and cosmology. To formulate the nature of 'gravitation' (so that from any given physical conditions equations relating the physical quantities may be deduced or vice versa), we systematize the laws of physics and the fundamental physical constants (of quantum electrodynamics and nucleon-meson dynamics [2, 3, 15]) through the singularity '**1**' in the *Principle of the One-and-the Many* and the Logarithmic Slide-Rule for Physical Relationships (LSPR) [1, 18, 19, 20].

3 The Principle of the One-and-the-Many

The Principle of the One-and-the-Many rests on the Principle of the Initial Conditions and the Principle of the Final Conditions of measurement wherein conceivable property of the *one* (individual quantity q_k) is also a property of the many (a number of q_k's: group quantity Q_k). If we regard a number of identical balls as many (Georg Cantor's *sets*), having a unity between them, then it is feasible to roll up the balls (or the *null, empty sets*) and mathematically unite them together, thereby moving from the many into the one. Indeed, *unity and multiplicity are two inverse views* of the same phenomena (Table 1). It is instructive to consider that in any *equilibrium*, it is impossible to have a group of balls without having individual balls and vice versa ([1, equation (2)] or [18, equation (4)]).

Note 1. The nature of space has dominated our thinking. Customarily, a discrete bundle of energy is called a quantum. Our work [1, 18] indicates that physical quantities are comprised of discrete bundles of the '**1**' (singularities): a number invariant q_k points of *empty space*: Final Equilibrium (Invariance) state. In suggested formalism, empty space (the Highest Common Factor) is an *invariant physical structure* with properties of its own. In addition, each singularity has two inverse points of view. That is, as in (1), the q_k of '**1**': the *point*: singularity, and the Q_k of '**1**': the *empty space*: Cantor's/Gödel's/Cohen's *Continuum* (where the universe and the laws of physics materialize: Plato's "receptacle, and in a manner the nurse, of all generation": Einstein's (1924) "Continuum which is equipped with physical properties; for the general theory of relativity") are *inverted viewpoints* of the same reality. Namely, the inverse of *one* zero dimension point (where the log of 1 is zero) is the *many* zero dimension points (where the inverse of log 1 is the *Absolute Infinity* [1, 18]. Between these two inverse landscapes encoded potential possibilities exist. That is, in the Principle of the One/Many, the individual phenomenon q_k is an inverted group phenomena Q_k, where

$$\mathbf{1} = Q_k q_k, \tag{1}$$

and q_k is either equal to, or less than ($\leq$) **1**, or **1** is equal to, or less than ($\leq$) Q_k, where

$$q_k \leq \mathbf{1} \leq Q_k. \tag{2}$$

The q_k and Q_k values are determined by physical constants. Consider the following illustrative examples, in Table 1, of individual and collective phenomena in the One-and-the-Many Principle (proposed new gravitation quantities are highlighted with bold letters):

Table 1. The "**1**" and the One-and-the-Many principle through the laws of physics.

$\boldsymbol{q_k}$**: Individual Quantity**	$\boldsymbol{Q_k}$**: Group Quantity**
Period of harmonic motion T	Frequency f $(= \mathbf{1}/T)$, where $Tf = \mathbf{1}$
Conductance G	Resistance R $(= \mathbf{1}/G)$, where $GR = \mathbf{1}$
Inductance L	Reluctance r $(= \mathbf{1}/L)$, where $Lr = \mathbf{1}$
Resistivity ρ	Conductivity σ $(= \mathbf{1}/\rho)$, where $\rho\sigma = \mathbf{1}$
Compton wavelength λ_c	Number of waves n $(= \mathbf{1}/\lambda_c)$, where $\lambda_c n = \mathbf{1}$
Magnetic flux quantum $\Phi_0 = h/2e$	Josephson constant $2e/h$, where $(\Phi_0)(2e/h) = \mathbf{1}$
Quantized Hall conductance e^2/h	von Klitzing constant $R_K = h/e^2$, where $(e^2/h)(R_K) = \mathbf{1}$
Gravitational penetrability z_0	Gravitational Constant G $(= \mathbf{1}/z_0)$, where $(z_0)(G) = \mathbf{1}$
Gravitational field quantum $\acute{\Gamma}$	Gravitational field strength g $(= \mathbf{1}/\acute{\Gamma})$, where $(\acute{\Gamma})(g) = \mathbf{1}$
Gravitational resonance cond. LC	**Gravitational potential density** ϑ $(= \mathbf{1}/LC)$, where $(LC)(\vartheta) = \mathbf{1}$

The singularity/Continuum are of *absolute uncertainty* (point of inversion: a natural location for the collapse of the wave function, while the mathematical formalism of the Heisenberg uncertainty relationships (expressed in terms of the building blocks of nature, i.e., energy/time) is of *relative uncertainty.* At a singularity the laws of science and our ability to predict, break down [1, 18]. The Heisenberg uncertainty relationships demonstrate the workings of a singularity, as expressed in equations (1) and (2). Similarly, because of the '**1**'/'=' in (1), a particle cannot be expressed by a wave packet, in which both the momentum and the position have arbitrary ranges. They must be scale-invariant [2, equations (36)–(40)]. As we make the range of one of them larger the range of the other becomes smaller, according to equation (1). Here the quantum uncertainty is not tied to one particular quantity but slides from quantum entity to quantum entity (the Principle of Position [18]: a rainbow appears at a different time in a different place with different intensity for *each* observer [19].

Physical quantities (or meaning), on the other hand, are created by *limits.* Namely, *meaning illustrates the unit interval between* ('**1**'/'=') *points* (singularities) [18]. To represent a physical quantity: natural unit-of-measurement (the Principle of Natural-Unit-of-Measurement [18]: Georg Cantor's non-empty set) in the Continuum *two* scale-invariant points in equilibrium ('**1**'/'=') are required. The unit interval (i.e., of open and closed lines, or Edward Witten's strings) between points (nodes) in the Continuum can be established via the fundamental physical constants, or with two quantities in terms of which a third quantity is described. For example, velocity is characterized in terms of m/sec. By analogy to mathematical zero's role, as a *shifter in value* (i.e., 83 to 80003), the '**1**'/'=' states in the Continuum are said to be quantized (natural grouping: the Principle of Quantization [18]). This means, ultimately the '**1**' (the Continuum) and the equality '=' states are scale-invariant, dimensionless, and quantized. It also suggests: Coleman et al. [21], *the invariance of the Continuum is the invariance of the Universe*, which is discussed elsewhere [18].

The concept of *dimension* is fundamental to all of mathematics and physics. With a series leaps of insight, the work of Euclid (defining a *point*); Eudoxus (introducing the concept of a *potential infinity*: enabling Newton, Leibniz, Gauss, Euler, and others approach zero/infinity, thus facilitating development of a *limit*); Bolzano–Weierstrass (showing that infinite sequences in a bounded space contain *limit points*); Galileo (leaping from potential infinity to *actual infinity*: an infinite set can be equal in number of elements to the smaller subset of itself); Cantor (arriving at *actual infinity*, and learning important truths about it, starting with sets); Peano (characterized as the empty set); Hahn–Banach (giving conditions where a linear functional can be extended to the full space that shares boundedness conditions with the functional); Zermelo (helping to design axioms of set theory), Gödel (proposing the incompleteness theorem); Cohen (concluding that the Continuum is beyond the lower infinities), and others, unlocked a door to the Absolute Infinity.

Cantor gave his sequence $\varkappa_0, \varkappa_1, \varkappa_2, \varkappa_3, \ldots$ of alephs (infinities) the name taf, π, to mean finality: every infinite cardinal had to be an aleph — belonging to the system π that includes all alephs. From the Principle of the One/Many, we see through the laws of physics, equation (1), and Table 1, a point and the Absolute Infinity are the inverse of each other. Using Cantor's sets, in the One/Many group, we have: $\varkappa_0 = 1/\pi$, where the point is '**1**' : $\varkappa_0$, and the Continuum is '**1**' : π. Therefore, as with zero/numbers, and numbers with one another, the suggested interpretation brings singularity, physical quantities and the fundamental physical constants closer together: changing our understanding of the quantities themselves. The observer's experience is expressed in the classical language of actualities (physical quantities), while the invariable/unmeasured quantum realm is not represented as a wavewise superposition of possibilities, or taf, π, but with the '**1**'. This offers a more general framework, than that provided by the standard interpretation for 'gravity' physics. In particular, when we consider the gravitational/mass relationships in nuclear/earth's dimensions, gravity no longer is gravity, because of scale invariance, becomes strong or weak force [1, equation (30)].

The LSPR operates on the same mathematical principle as a logarithmic slide-rule for numbers [1, 19, 20]. A logarithmic slide-rule has zero/numbers. A LSPR has the '**1**' and physical quantities. The LSPR generates experimentally verified equations of the laws of physics and fundamental physical constants. Furthermore, a LSPR will, in simple equations, give form to (i.e., predict) new (unknown: 'hidden' variables, Bohm [22]) physical relationships/constants. A more comprehensive discussion on the Principle of the Initial Conditions ('**1**'), the Principle of the One-and-the-Many, and the LSPR, can be found in [1, 18]. Conventional symbols and the SI units (in which they are usually quoted) are deployed.

Measuring a physical quantity signifies comparing the quantity with a standard quantity (unit of measurement) of the same scale and nature. To make the relationships more accessible to physicists who work with gravitational phenomena, model building, and to facilitate new/more encompassing gravitational experiments and precision measurements (of the basic physical relations/constants), we restate the meaning of these equations by providing several relationships for the same phenomena. These equations advance the development through direct observation of new experimental investigations in gravity, enabling one to formulate additional systems of units and their conversion factors of physics.

4 The gravitational field strength $\boldsymbol{g}$

The gravitational field strength $\boldsymbol{g}$ $(= \boldsymbol{F}/m)$, in $\mathrm{m \cdot s^{-2}}$, at a point of the gravitational *force* $\boldsymbol{F}$, in N, per unit mass m, in kg, can be written as the *angular gravitational potential* [1, equation (37)]

$$\boldsymbol{g} = V_g/S, \tag{3}$$

where V_g is the gravitational potential, in $\mathrm{J \cdot kg^{-1}}$, and S is the *length* unit, in $\mathrm{m \cdot rad^{-1}}$.

Note 2. The unit radian (rad), for plane angle, has historically been designated as a supplementary unit [1]. In 1980, the International Committee for Weights and Measures determined that the unit radian and steradian are equivalent to the number one 1 and may be omitted in the expression for derived units. For completeness of presentation, due to the angle of rotation, expressed in radiant per cycle, is a physical quantity, which like other quantities enters into physical relationships, it is included here. Furthermore, as stated earlier, to represent a quantity, two dimensionless points are necessary.

Notice, deploying constants from [2] in equation (2), the $\boldsymbol{g}$ and V_g quantities, in equation (3), for the electron, are larger than '**1**', therefore, they are Q_{kq} quantities. Additionally, the S quantity, for the electron in [2], is smaller than '**1**', therefore, it is a q_k quantity. To make equation (3) for students more logical, we could describe (3) in Q_{kq} terms (i.e., $\boldsymbol{g} = V_g$ times the number of separation points $1/S$), or as a quantum q_k expression.

Therefore, following this approach, we can write equation (3) as a relationship of the *gravitational flux density* $\boldsymbol{M}$, where $\boldsymbol{M} = m/S^2$, $\mathrm{kg \cdot m^{-2} \cdot rad^{-1}}$, and the *Newtonian constant of gravitation* G, in $\mathrm{m^3 \cdot kg^{-1} \cdot s^{-2}}$, whereas

$$\boldsymbol{g} = \boldsymbol{M}G. \tag{4}$$

Moreover, entering equation (4) as a relationship of *pressure* P, in Pa, and the gravitational flux density, gives us

$$\boldsymbol{g} = P/\boldsymbol{M}. \tag{5}$$

Combining equations (4) and (5), provides

$$\boldsymbol{g} = (GP)^{1/2}. \tag{6}$$

5 The gravitational constant G

The Gravitational Constant G, in $\mathrm{m^3 \cdot kg^{-1} \cdot s^{-2}}$, is routinely stated in terms of the gravitational force $\boldsymbol{F}$, that two particles of masses m_1 and m_2 separated by a distance S exert on each other, where $\boldsymbol{F} = Gm_1m_2/S^2$. We characterize the universal constant G by way of the gravitational field strength $\boldsymbol{g}$ and the gravitational flux density $\boldsymbol{M}$, as shown in equation (4), $G = \boldsymbol{g}/\boldsymbol{M}$. The Newtonian constant of gravitation G can also be given in terms of the gravitational potential V_g and the *linear mass density* μ ($= m/S$), where the mass per-unit-length of μ is in $\mathrm{kg \cdot m^{-1}}$. Thus,

$$G = V_g/\mu. \tag{7}$$

This then leads us to a third method representing equation (7) using the length S and the *gravitance* $\Omega = m/V_g$, in $\mathrm{kg^2 \cdot J^{-1}}$, equation (21). Accordingly,

$$G = S/\Omega. \tag{8}$$

Combining equations (4) with (5) gives

$$G = P/\boldsymbol{M}^2 = 1/\chi\boldsymbol{M}^2, \tag{9}$$

whereby χ is compressibility in $\mathrm{m^2 \cdot N^{-1}}$.

Note 3. The Newtonian constant of universal gravitation is the constant of proportionality within an equation relating to the attraction force between any two bodies (particles) separated by distance S. In a scale-invariant setting of nuclear dimensions, *transformation* of physical quantities and scale changes (renormalizability) take place [1, 3, 15, 16]. The dimensional values of quantities (i.e., G, S, m, $\boldsymbol{F}$) are no longer gravitational values of the Earth, but nuclear values. This can be seen, by defining the gravitational constant G from the law of periods.

Note 4. As suggested earlier, the Continuum is invariant. Separation between points, in the Continuum, determines the meaning of the natural unit-of-measurement (i.e., physical quantity) [18], making the laws of physics invariant. Considering Note 3 and the lack of scale-invariance in general relativity [18], reduce the general relativity to a limited construct, that is, says Hoyle, Burbidge and Narlikar [23]: "the equations of general relativity are not scale-invariant. They are the special form to which the scale-invariant equations reduce with respect to a particular scale, namely that in which particle masses are everywhere the same".

6 The gravitational potential V_g

The Gravitational Potential V_g, at a point, is the potential energy per unit test mass, in $\mathrm{J \cdot kg^{-1}}$. The gravitational potential is usually determined using $V_g = -Gm/S$. The V_g can also be depicted as the linear stopping power, where $V_g = \boldsymbol{g}S$. Furthermore, the gravitational potential can be expressed as an *area* $\boldsymbol{A}$, in $\mathrm{m^2}$, and the *angular speed* (rotation rate) ω, in $\mathrm{rad \cdot s^{-1}}$, where

$$V_g = \boldsymbol{A}\omega^2. \tag{10}$$

Equation (10) is, in addition related to purely electric and magnetic quantities by

$$V_g = 1/\varepsilon_0\mu_0. \tag{11}$$

For ε_0 and μ_0 terms see the gravitational penetrability, in Section 7. Equation (10) can also be characterized by the gravitational field strength and the *gravitational potential density* ϑ, in $\mathrm{rad^2 \cdot s^{-2}}$ (Section 10), where

$$V_g = \boldsymbol{g}^2/\vartheta. \tag{12}$$

Also,

$$V_g = \boldsymbol{g}^2/\omega^2. \tag{13}$$

7 The gravitational penetrability

Analogous to the (electric) permittivity of vacuum ε_0 $(= C/S)$, in $\mathrm{F \cdot m^{-1}}$, where C is the capacitance in farad (F), and the (magnetic) permeability of vacuum μ_0 $(= L/S)$, in $\mathrm{H \cdot m^{-1}}$, where L is the inductance in henry (H), we propose a new physical quantity, the (gravitational) *penetrability of free space* (vacuum) z_0 $(= \Omega/S)$, equation (22), expressed in $\mathrm{kg \cdot s^2 \cdot m^{-3}}$. The gravitational penetrability is the inverse of the gravitational constant G

$$z_0 = 1/G. \tag{14}$$

Combining the angular rotation rate and the density d $(= m/V_0)$, in $\mathrm{kg \cdot m^{-3}}$, where V_0 is volume, in $\mathrm{m^3}$. We can write equation (14) as

$$z_0 = d/\omega^2. \tag{15}$$

Equation (14) can also be stated in terms of the mass m, gravitational potential V_g, and the separation S, where

$$z_0 = m/V_gS. \tag{16}$$

Or, in terms of work W, in J, volume and the gravitational field strength,

$$z_0 = W/V_0g^2. \tag{17}$$

8 The gravitational flux density

Comparable to the electric flux density $\boldsymbol{D}$ $(= \varepsilon_0 \boldsymbol{E})$, in $\mathrm{C \cdot m^{-2}}$, and the magnetic flux density $\boldsymbol{B}$ $(= \mu_0 \boldsymbol{H})$, in T $(\mathrm{Wb \cdot m^{-2}})$, we derived the *gravitational flux density* $\boldsymbol{M}$, in $\mathrm{kg \cdot m^{-2} \cdot rad^{-1}}$, where

$$\boldsymbol{M} = z_0 \boldsymbol{g}. \tag{18}$$

Equation (18) can be represented as a relationship of the pressure and the gravitational field strength, $\boldsymbol{M} = P/\boldsymbol{g}$, or the gravitational field strength and the gravitational constant G, where $\boldsymbol{M} = \boldsymbol{g}/G$. In addition, the gravitational flux density can be given by the mass m and a unit of length S, where,

$$\boldsymbol{M} = m/S^2. \tag{19}$$

Equation (19) can also be described in terms of the Hooke's law proportionality *spring constant* k, in force per unit length, and the gravitational potential, where

$$\boldsymbol{M} = k/V_g. \tag{20}$$

9 The gravitance

Comparable to the capacitance $(C = e/V)$ in the electric domain and the inductance $(L = \phi/i)$ in the magnetic domain we suggest the *gravitance* Ω, stated in $\mathrm{kg^2 \cdot J^{-1}}$, in the gravitational domain to be:

$$\Omega = m/V_g, \tag{21}$$

where e is the electric flux (elementary charge), in C, ϕ is the magnetic flux, in Wb, V is the electric potential, in V, and i is the magnetic potential, in A. The gravitance can also be found from (8), where $\Omega = S/G$. Combining (8) with (14) we obtain,

$$\Omega = S z_0. \tag{22}$$

Also, the gravitance can be written by uniting equations (11) and (21), where

$$\Omega = \varepsilon_0 \mu_0 m. \tag{23}$$

10 The gravitational potential density

Comparable to the magnetic potential (current) density $\boldsymbol{j}$ $(= i/\boldsymbol{A})$, in $\mathrm{A \cdot m^{-2}}$, and the electric potential (voltage) density $(= V/\boldsymbol{A})$, in $\mathrm{V \cdot m^{-2}}$, we suggest the *gravitational potential density* ϑ, expressed in $\mathrm{rad^2 \cdot s^{-2}}$, where

$$\vartheta = V_g/\boldsymbol{A}. \tag{24}$$

Equation (24) can, in addition, be characterized via the gravitational field strength and S, where

$$\vartheta = \boldsymbol{g}/S. \tag{25}$$

Furthermore, equation (25) can be written as the time t, in $\mathrm{s \cdot rad^{-1}}$, where

$$\vartheta = 1/t^2. \tag{26}$$

We can also find the gravitational potential density through the gravitational flux density and gravitance

$$\vartheta = \boldsymbol{M}/\Omega. \tag{27}$$

11 The gravitational force

Analogous to the electromagnetic field which exerts sideways electric $\boldsymbol{F}_y = VS \times \boldsymbol{D}$ and magnetic $\boldsymbol{F}_z = iS \times \boldsymbol{B}$ forces, we suggest that a sideways *gravitational force* $\boldsymbol{F}_x$, in N, is exerted in the gravitational field subjected to the gravitational flux density, whereby,

$$\boldsymbol{F}_x = V_g S \times \boldsymbol{M}, \tag{28}$$

where, $VS = e/\varepsilon_0$; $iS = \phi/\mu_0$; and $V_g S = m/zo = mG$. For a second approach, consider that $\boldsymbol{F}_y = e\boldsymbol{H}$, $\boldsymbol{F}_z = \phi\boldsymbol{H}$, then

$$\boldsymbol{F}_x = m\boldsymbol{g}, \tag{29}$$

where the electric field strength $\boldsymbol{E}$ is in $\mathrm{V \cdot m^{-1}}$, and the magnetic field strength $\boldsymbol{H}$ is in $\mathrm{A \cdot m^{-1}}$. Notice that the electric, magnetic and gravitational forces are in equilibrium at singularity ('='): $\boldsymbol{F}_y = \boldsymbol{F}_z = \boldsymbol{F}_x$. Further, in electromagnetic traveling waves when the lines of $\boldsymbol{E}$ are parallel to the y-axis, and the lines of $\boldsymbol{B}$ are parallel to the z-axis, we suggest that, the lines of $\boldsymbol{M}$ are parallel to the x-axis. Our observation indicates that $\boldsymbol{E}$, $\boldsymbol{B}$ and $\boldsymbol{M}$ are perpendicular to one another. In addition, consistent with equation (11) $V_g = 1/\varepsilon_0\mu_0$, and therefore is on the velocity axis. Furthermore, $\boldsymbol{E}$, $\boldsymbol{B}$ and $\boldsymbol{M}$ are in phase (they achieve their maxima at the identical time, and they are zero at the same time).

12 The gravitational resonance condition

In our discussion of resonances, a damped LC circuit oscillating at natural frequency $\omega = (LC)^{-1/2}$, is described in terms of gravitational quantities as the *gravitational resonance condition*. We derived the LC condition by way of the gravitance and the gravitational flux density, where

$$LC = \Omega/\boldsymbol{M}. \tag{30}$$

Also, equation (30) can be written as a relationship of the area and the gravitational potential [1, equation (23)],

$$LC = A/V_g. \tag{31}$$

In addition, equation (30) can be expressed by the gravitational potential and the gravitational field strength, where

$$LC = V_g/\boldsymbol{g}^2. \tag{32}$$

Combining equations (31) with (32) gives

$$LC = S/\boldsymbol{g}. \tag{33}$$

Furthermore,

$$LC = 1/\vartheta. \tag{34}$$

13 The gravitational field quantum

The One-and-the-Many Principle, can be utilized with the gravitational field strength $\boldsymbol{g}$ to find the *gravitational field quantum* $\acute{\Gamma}$, in $\mathrm{s}^2 \cdot \mathrm{m}^{-1}$, where

$$\acute{\Gamma} = 1/\boldsymbol{g}. \tag{35}$$

Earlier we expressed the gravitational field strength in Q_k language. Equation (36) is a *quantum* q_k expression, where the gravitational field quantum equals the *gravitational potential quantum* $(1/V_g)$ times the separation quantum S,

$$\acute{\Gamma} = (1/V_g)S. \tag{36}$$

Additionally, the gravitational field quantum $\acute{\Gamma}$ and the velocity v can be used to describe the time quantum t, we find

$$t = \acute{\Gamma} v. \tag{37}$$

For the electron $t = 1.1812 \times 10^{-22}$ seconds^2. Derivation/higher accuracy constants can be found in [2, equation (10)]. In Tables 2, 3 and 4 we list gravitational (x), electric (y) and magnetic (z) correlation between the quantities and their relationships.

Table 2. Symmetries of electric, magnetic and gravitational quantities.

1	$\boldsymbol{F}/e =$ Electric field strength	$\boldsymbol{E}$
	$\boldsymbol{F}/\phi =$ Magnetic field strength	$\boldsymbol{H}$
	$\boldsymbol{F}/m =$ Gravitational field strength	$\boldsymbol{g} = S/LC$
2	$W/e =$ Electric potential	$V = \boldsymbol{E}S$
	$W/\phi =$ Magnetic potential	$i = \boldsymbol{H}S$
	$W/m =$ Gravitational potential	$V_g = \boldsymbol{g}S = A/LC$
3	$e/A =$ Electric flux density	$\boldsymbol{D} = \boldsymbol{Y}C$
	$\phi/A =$ Magnetic flux density	$\boldsymbol{B} = \boldsymbol{J}L$
	$m/A =$ Gravitational flux density	$\boldsymbol{M} = \vartheta\Omega = \Omega/LC$
4	$V/A =$ Electric potential (voltage) density	$\boldsymbol{Y} = \boldsymbol{E}/S$
	$i/A =$ Magnetic potential (current) density	$\boldsymbol{J} = \boldsymbol{H}/S$
	$V_g/A =$ Gravitational potential (m) density	$\vartheta = \boldsymbol{g}/S = 1/LC$
5	$e/V =$ Capacitance	$C = e^2/W = \boldsymbol{D}/\boldsymbol{Y}$
	$\phi/i =$ Self inductance	$L = \phi^2/W = \boldsymbol{B}/\boldsymbol{J}$
	$m/V_g =$ Gravitance	$\Omega = m^2/W = \boldsymbol{M}LC = \boldsymbol{M}/\vartheta$
6	$\boldsymbol{F}/\boldsymbol{E} =$ Electric flux	$e = W/V$
	$\boldsymbol{F}/\boldsymbol{H} =$ Magnetic flux	$\phi = W/i$
	$\boldsymbol{F}/\boldsymbol{g} =$ Gravitational flux (mass)	$m = W/V_g$

Table 3. Symmetries of gravitational ($\boldsymbol{x}$), electric ($\boldsymbol{y}$) and magnetic ($\boldsymbol{z}$) quantities.

	x	y	z
1	$\Omega = m/V_g$	$C = e/V$	$L = \phi/i$
2	$zo = \Omega/S$	$\varepsilon_0 = C/S$	$\mu_0 = L/S$
3	$\boldsymbol{M} = z_0\boldsymbol{g}$	$\boldsymbol{D} = \varepsilon_0\boldsymbol{E}$	$\boldsymbol{B} = \mu_0\boldsymbol{H}$
4	$G = \boldsymbol{g}/\boldsymbol{M}$	$1/\varepsilon_0 = \boldsymbol{E}/\boldsymbol{D}$	$1/\mu_0 = \boldsymbol{H}/\boldsymbol{B}$
5	$\boldsymbol{F}_x = m\boldsymbol{g}$	$\boldsymbol{F}_y = e\boldsymbol{E}$	$\boldsymbol{F}_z = \phi\boldsymbol{H}$

14 Results and discussion

One of the most significant advances in the field of physics was the scientific method: the procedure physicist use to gain knowledge. To quantify the experiment's results, measurement of phenomena has been essential to the scientific method. We suggest comparatively simple systematic

Table 4. Comparison of the electron, proton, neutron, and earth calculations and constants.

Quantity	Symbol	Earth	Electron	Proton	Neutron	Units
Mass	m	5.972×10^{24}	9.109×10^{-31}	1.673×10^{-27}	1.675×10^{-27}	kg
Gravitational constant	G	6.674×10^{-11}	3.494×10^{33}	7.922×10^{29}	7.909×10^{29}	$\mathrm{m^3 \cdot kg^{-1} \cdot s^{-2}}$
Gravitational field strength	g	9.807	2.538×10^{30}	1.262×10^{27}	1.260×10^{27}	$\mathrm{m \cdot s^{-2}}$
Gravitational potential	V_g	6.252×10^{7}	8.988×10^{16}	1.293×10^{15}	1.292×10^{15}	$\mathrm{J \cdot kg^{-1}}$
Gravitational penetrability	z_0	1.498×10^{10}	2.862×10^{-34}	1.262×10^{-30}	1.264×10^{-30}	$\mathrm{kg \cdot s^2 \cdot m^{-3}}$
Gravitational flux density	M	1.469×10^{11}	7.265×10^{-4}	1.593×10^{-3}	1.593×10^{-3}	$\mathrm{kg \cdot rad^2 \cdot m^{-2}}$
Gravitance	Ω	9.553×10^{16}	1.014×10^{-47}	1.294×10^{-42}	1.296×10^{-42}	$\mathrm{kg^2 \cdot J^{-1}}$
Gravitational potential density	ϑ	1.538×10^{-6}	7.1673×10^{43}	1.232×10^{39}	1.229×10^{39}	$\mathrm{rad^2 \cdot s^{-2}}$
Gravitational force	F_x	5.857×10^{25}	2.312×10^{0}	2.111×10^{0}	2.110×10^{0}	N
Gyroradius	S	6.378×10^{6}	3.541×10^{-14}	1.025×10^{-12}	1.025×10^{-12}	$\mathrm{m \cdot rad^{-1}}$
Gravitational field quantum	$\acute{\Gamma}$	1.019×10^{-1}	3.940×10^{-31}	7.924×10^{-28}	7.937×10^{-28}	$\mathrm{s^2 \cdot m}$

treatment/methodology of how, by unifying *theory* (algebra of logical and measurable evidence) with *measurement* (known fundamental constants/laws of physics), and an indirect procedure of physical constant and fundamental quantity formation (symmetry, scale-invariance, et al.), improved understanding in the observation/measurement, the formulation of physical laws and the development of a theory that is used to predict new phenomena can yield otherwise unobtainable results. Fortunately, the *Continuum is invariant* '**1**'. Known laws of physics and fundamental physical constants (obtained through high precision measurements of the Continuum, i.e., separation between '**1**' and '=') are reducible to mathematical relations/operations, which are *constant*, and which can be used to penetrate, define, calculate, and predict more accurate measurements [2] of fundamental quantities (Table 2), values of the physical constants, '*a priori*' numerical computations, discovery of new phenomenon, and looking or thinking about a problem in a totally different way.

Based on these elementary considerations and systematic procedures, one of the important objects of this note consists in suggesting very simple formulas, physical relationships, fundamental constants, and experimental tests for gravity physics. As far as we know, there are less than 5000 physical relationships that have been verified, in the last 400 years by science. Using five constants [2], computers, and the suggested methodology, we obtained over 100,000 physical relationships, and more than 20 new physical quantities, some of which are presented here as gravity physics. Whether the formulas, elucidation of their properties, and correlation depicted in the present note is consistent with experimental facts is an open question. However, the approach is based on experimental data (known constants and laws of physics) and the agreement of the values in Appendix A and B is very strong evidence in support of this methodology of *measurement-based, mathematical procedure* for obtaining these results. Also, it might be pointed out, the remarkable agreement between other analogous formulas generated in similar manner [2, 4], and the experiments, can leave but little doubt that the suggested phenomenological relationships constitute gravity phenomena.

Further, the experimental support of the approach indicates very convincingly that the integration of a scattered and immense body of fundamental physical phenomena into a more systematic order is possible [1, 19, 20]. It should be noted, at the present stage of physics we are not able to predict accurately new gravitational quantities or fundamental constants of nucleon-meson dynamics [3]. Should experiments corroborate the suggested relationships/constants, physicist would gain a phenomenological leap in our understanding of the Continuum '**1**', through a simple mathematical method for the measurements of phenomena, formulation of physical laws from the generalization of the phenomena and the development of theories that is used to predict new phenomena. Furthermore, because the Continuum is invariant and there are infinite potentialities within it, measurements between '**1**' and '=' seems likely to continue (characterizing fundamental quantities/constants, revealing hitherto ignored physical effects, inducing *inverted* populations to radiate in concert (through the Principle of One/Many), i.e., generate coherent states of the gravitational field, laser technology, et al.), I see no let up in the bread-and-butter business of measuring and theoretical prediction of phenomena. Conversely, I expect accelerated advancement and greater opportunities in all branches of science.

Appendix A. Sample derivations and calculations

In [2] we have provided the derivation of the fundamental constants in quantum electrodynamics, and in [3] the fundamental physical constants of nucleon-meson dynamics. Now, we will discuss the derivation of Earth's dimensions.

To obtain the standard acceleration of gravity $\boldsymbol{g}$ we utilized the 1998 CODATA (the Committee on Data for Science and Technology, the international arbiter of metrology) set of recommended values of the basic constants and conversion factors of physics [24]. The preliminary value of G comes from higher precision measurement of the gravitational constant by Jens Gundlach and Stephen Merkowitz at the University of Washington, (Seattle), reported at APS April 2000 meeting in Long Beach (Ca) [25]. The gravitational flux density $\boldsymbol{M}$ is a derivative of $\boldsymbol{M} = \boldsymbol{g}/G$, while the gravitational penetrability, of equation (14), is $z_0 = 1/G$.

The Earth has an equatorial radius of 6.378×10^6 m, a polar radius of 6.357×10^6 m, and a mean radius of 6.371×10^6 m. The Earth's gravitational field (polar surface gravity) varies from place to place on it's surface, with the main variation occurring with latitude, averaging approximately $9.8322\ \mathrm{m \cdot s^{-2}}$at the poles, and at the Equator (equatorial surface gravity) $9.7303\ \mathrm{m \cdot s^{-2}}$ (includes rotation). We deployed the standard acceleration of gravity value $9.80665\ \mathrm{m \cdot s^{-2}}$, the Earth mass m of $5.97223(\pm 0.00008) \times 10^{24}$ kg, and the Newtonian constant of gravitation values of G $6.674215 \pm 0.000092 \times 10^{-11}\ \mathrm{m^3 \cdot kg^{-1} \cdot s^{-2}}$, to find the Earth's value of the length of the semi-major axis S, via $S^2 = m/\boldsymbol{M}$ and $\boldsymbol{M} = \boldsymbol{g}/G$. This method yields the length of the semi-major axis of S 6.37541×10^6 m. The length of the semi-major axis can customarily be determined by the arc method. The determinations of dimensions of the Earth ellipsoid from arc measurements yield 6.37816×10^6 m for the semi-major axis. Similarly, utilizing the G, $\boldsymbol{g}$, and the Earth ellipsoid from arc measurements, of 6.37816×10^6 m value in equations (19) and (4), where $m = S^2\boldsymbol{g}/G$, we attain 5.97734×10^{24} kg for the Earth's mass. Because we were studying the same problem from two different points, for the two approaches to be compatible, the present measurements (of the length of the semi-major axis or the Earth's mass) could be refined, i.e., $\boldsymbol{g}$, G, and S measurements would have to use identical initial conditions ('**1**').

Appendix B. Comparison of the electron, proton, neutron, and earth calculations and constants

For comparative purposes we have computed the electron [2], proton and neutron [3] length units. Notice the proton and the neutron length of the semi-major axis S is approximately 30 times larger than that of the electron.

The gravitance value is obtained via equation (22) ($\Omega = Sz_0$), the gravitational potential via ($V_g = S\boldsymbol{g}$), and the gravitational potential density by equation (25) ($\vartheta = \boldsymbol{g}/S$). In Table 4 we list values of the electron, proton, neutron, and Earth calculations and constants, as they relate to 'gravitational' relationships.

[1] Bedrij O., Scale-invariance, unifying principle, order and sequence of physical quantities and fundamental constants, *Proc. of the Nat. Acad. of Sci. of Ukr.*, 1993, N 4, 67–73.

[2] Bedrij O., Fundamental constants in quantum electrodynamics, *Proc. of the Nat. Acad. of Sci. of Ukr.*, 1993, N 3, 40–45.

[3] Bedrij O. and Fushchych W.I., Fundamental constants of nucleon-meson dynamics, *Proc. of the Nat. Acad. of Sci. of Ukr.*, 1993, N 5, 62–65.

[4] Fushchych W.I. and Nikitin A.G., Symmetries of Maxwell's equations, Dordrecht, D. Reider Publishing Company, 1987.

[5] Lie S., Transformationgruppen, in 3 Bd., Leipzig, 1883;
Lie S. and Engel F., Theorie der Transformationgruppen, Leipzig, Teubner, 1988, 1890, 1893.

[6] Lorenz G.A., Electromagnetic phenomena in a system moving with any velocity smaller than that of light, *Proc. of Acad. of Sci.*, 1904, V.6, 809–830.

[7] Einstein A., Zur Electrodynamics bewegter Korper, *Ann. L. Phys.*, 1905, V.17, 891.

[8] Poincare H., Sur la dynamique de l'electron, *Comptus Rendus de Academie de Science*, 1905, V.140, 1504–1506; Sur la dynamique de l'electron, *Rediconti Cir. Mathematical Palermo*, 1906, V.21, 129–160.

[9] Heaviside O., On the forces, stresses and fluxes of energy in electromagnetic field, *Phil. Trans. Roy. Soc. A*, 1893, V.183, 423–480.

[10] Bateman H., On the conformal transformations of the space of four dimensional and their applications to geometric optics, *Proc. of London Math. Soc.*, 1909, V. 7, 70–92; The transformation of electrodynamical equations, *Proc. of London Math. Soc.*, 1909, V.8, 223–264.

[11] Cuningham E., The princeple of relativity in electrodynamics and an extension thereof, *Proc. of London Math. Soc.*, 1909, V.8, 77–98.

[12] Rainich G.J., Electrodynamics in the general relativity theory, *Trans. Am. Math. Soc.*, V.27, 106–136.

[13] Weinberg S., The quantum theory of fields. Vol. III. Supersymmetry, Cambridge, Cambridge University Press, 2000.

[14] Fushchych W.I. and Nikitin A.G., Symmetries of equations of quantum mechanics, New York, Allerton Press, Inc., 1994.

[15] Bedrij O. and Fushchych W.I., On the electromagnetic structure of elementary particles masses, *Proc. of the Nat. Acad. of Sci. of Ukr., Ser. A*, 1991, N 2, 38–40.

[16] Bedrij O. and Fushchych W.I., Plank's constant is not constant in different quantum phenomena, *Proc. of the Nat. Acad. of Sci. of Ukr.*, 1995, N 12, 26–27.

[17] Bedrij O., Connection of π with the fine structure constants, *Proc. of the Nat. Acad. of Sci. of Ukr.*, 1994, N 10, 63–66.

[18] Bedrij O., Revelation and verification of ultimate reality and meaning through direct experience and the laws of physics. Ultimate reality and meaning, Toronto, University of Toronto Press, 2000, 23:1.

[19] Bedrij O., Grand unification of the science of physics through the cosmolog, in Amer. Assoc. for the Adv. of Sci. Abstracts, Annual Meeting, 1990.

[20] Institute of Mathematical Physics, Landmark Breakthrough in Physics, Physics Today, College Park, MD, American Institute of Physics (Prod. advert.), 1994.

[21] Coleman S., The invariance of the vacuum is the invariance of the world, *J. Math. Phys.*, 1966, V.7, 787.

[22] Bohm D., A suggested interpretation of the quantum theory in terms "hidden" variables. I, in The Physical Review, Woodbury, New York, American Institute of Physics, 1952, 166–179.

[23] Hoyle F., Burbidge G. and Narlikar J.V., A different approach to cosmology, Cambridge, Cambridge University Press, 2000.

[24] Mohr P.J. and Taylor B.N., The fundamental physical constants, in Physics Today, College Park, MD, American Institute of Physics, 2000.

[25] Schwarzschild B., Our knowledge of G gets worse, then better, in Physics Today, College Park, MD, American Institute of Physics, 2000, 21–21.