

Explicit reduction of completely integrable Hamiltonian systems with the full momentum additional integral

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The talk is devoted to the problem of exact reduction of some special Hamiltonian systems. Suppose the natural Hamiltonian system with the Hamiltonian

$$\mathcal{H} = \frac{1}{2} \sum_{i=1}^n p_i^2 + W(x_1, x_2, \dots, x_n)$$

is integrable by Liouville and admits the first integral (the full momentum) $\mathcal{P} = \sum_{i=1}^n p_i$.

The following problem arises: is it possible to state that the reduced (in regard to \mathcal{P}) Hamiltonian system (now with $n - 1$ degrees of freedom) is also integrable by Liouville? The equivalent formulation is: *does this system also admit a full collection of integrals containing the full momentum \mathcal{P} ?*

In this talk we investigate this problem in the class of Hamiltonian systems, which possesses the full collection of first integrals, which are polynomial in the momenta. Using some linear canonical transformation the Hamiltonian \mathcal{H} can be modified into

$$\mathcal{H} = \frac{1}{2} \sum_{i=1}^n p_i^2 + W(x_1, x_2, \dots, x_{n-1}), \quad (1)$$

whereas the full momentum is now p_n .

The necessary conditions of the existence of the full collection of involutory integrals in the form $(\mathcal{H} = F_1, F_2, \dots, F_{n-1}, F_n = P)$ for the Hamiltonian system (1) are investigated. The similar statement for the Hamiltonian (1) with a full collection of the first integrals being analytical in the momenta (at least one of them) is still open.

References

- [1] Moser J. Various aspects of integrable Hamiltonian systems, Progress in Mathematics, 8, Dynamical Systems. Birkhauser, 1980, 233–289.
- [2] Perelomov A.M. Integrable System of Classical Mechanics and Lie Algebras. I, Birkhauser, Basel 1990.