Asymptotic expansions for one-soliton solutions to perturbed Korteweg-de Vries equation

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Asymptotical method is one of the effective method for studying nonlinear differential equations. The idea of representation a solution to regularly perturbed differential equation belongs to Lagrange and was extensively used while studying problems of the celestial mechanics.

It is well known during researching different physical phenomena and processes, in particular, relaxational oscillations, the problem of studying singularly perturbed systems is arised. One of the first paper concerning this problem was one by A.N.Tikhonov ("On dependence solutions to differential equations on a small parameter" // Matematichskiy zbornik. – 1948. – Vol.22 (69), N.2. – P. 193 – 204 (In Russian)) the ideas of that were later developed by his followers [1] to construct through boundary functions asymptotical solution to the so-called Tikhonov system.

On the other hand, while describing processes of diffusion waves, shallow water, gas dynamics and others, a problem of studying perturbed partial differential equations appears as well as a problem of constructing their soliton-type solutions. It should be mentioned here that asymptotical approaches were succesfully applied for finding approximate solutions of the problem (see, for example, a number of papers by V.P.Maslov, G.A.Omel'yanov and S.Yu.Dobrokhotov [2-4] devoted to the problem).

We consider Korteweg-de Vries equation

$$u_{xxx} = a(x,\varepsilon)u_t + b(x,\varepsilon)uu_x,$$

where functions $a(x,\varepsilon)$, $b(x,\varepsilon)$ are given as follows

$$a(x,\varepsilon) = \frac{1}{\varepsilon^{N_0}}\sum_{k=0}^{\infty} a_k(x)\varepsilon^k, \qquad b(x,\varepsilon) = \frac{1}{\varepsilon^{N_0}}\sum_{k=0}^{\infty} b_k(x)\varepsilon^k.$$

Here $N_0 > 0$ is some natural, $x \in \mathbf{R}^1$, $t \in [0;T]$; $\varepsilon > 0$ is a small parameter.

The coefficients $a_k(x), b_k(x)$, $k = 1,\ldots,\infty$, are also supposed to be infinitely differentiable functions of variable $x \in \mathbf{R}^1$.

Using the ideas of perturbation theory we propose algorithm for constructing asymptotic soliton type solutions to the problem under the consideration.

Theorems on the order of asymptotical approximations are proved.

1. Vasil'eva A. B., Butuzov V. Ph. Asymptotical methods in the theory of singular perturbations. – M.: Vishaya shkola, – 1980. – 208 p.

2. Maslov V.P., Phedoryuk M.V. Quasiclassical approximation for equations of quantum mechanics. – Moscow: Nauka, – 1976. – 296 p.

3. Omel'yanov G.A. Interaction of waves of different scales in gas dynamics // Mathematical notes. – 1993. – Vol.53, N.1. – P. 148 – 151.

4. Dobrokhotov S.Yu. Hugoniot-Maslov chains for solitary vortices of the shallow water equations // Russian J. Math. Phys. – 1999. – Vol.6, N.2. – P. 137 – 173.