Nonlocal invariant reductions of supersymmetric hierarchies

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The supersymmetric generalizations for Lax integrable nonlinear dynamical system hierarchies in the form of smooth vector fields $K_j : M^{p|q} \to T(M^{p|q})$ on a functional supermanifold $M^{p|q} \subset C^\infty(\mathbf{S}^1 \times \Lambda_1; \Lambda_0^p \times \Lambda_1^q)$ such as:

$$(a, w)^T_{t_j} = K_j[a, w], \quad j \in \mathbf{N},\ t_j \in \mathbf{R}, \tag{1}$$

where $(a, w)^T \in M^{p|q}$ are even and odd superfunctions correspondingly, $a = a(x, \theta) = a_0(x) + \theta a_1(x)$, $w = w(x, \theta) = w_1(x) + \theta w_0(x)$, $x \in \mathbf{S}^1$, $\theta^2 = 0$, $\Lambda = \Lambda_0 \oplus \Lambda_1$ being the Grassmann algebra, obtained as Hamiltonian flows on the Lie superalgebra of pseudo-superdifferentilal operators with infinite set of local conservation laws $\gamma_n \in D(M^{p|q})$, $n \in \mathbf{N}$, which are owning a matrix Lax type representations, are considered. The latters are the compatibility conditions for the first order linear differential equations:

$$D_\theta Y = A[a, w; \lambda]Y, \tag{2}$$

$$Y_{t_j} = B_j[a, w; \lambda]Y, \tag{3}$$

where $j \in \mathbf{N}$ and $\lambda \in \mathbf{C}$ is an invariant with respect to (1) parameter.

Via the procedure of adding the nonlinear dynamical systems (1) by evolution relationships for eigenfunctions and adjoint ones of the spectral problem (2) and by means of the Bogoyavlensky-Novikov type reduction method the differential-geometric structure of the invarint finite-dimensional supersubmanifolds $M_n^{p|q} \subset M^{p|q}$:

$$M_n^{p|q} = \{(a, w)^T \in M^{p|q} :\ \operatorname{grad} L_n = 0\}, \quad L_n = -\gamma_n + \sum_{k=1}^{N} c_k \lambda_k, \tag{4}$$

where λ_k are some eigenvalues of (2), $c_k \in \mathbf{C}$, $k = \overline{1, N}$, and the Lax-Liouville integrability problem for the vectors fields (3) on $M_n^{p|q}$ are investigated. Namely, the formulas to find the even supersymplectic form on $M_n^{p|q}$ and Hamiltonian functions for (3) are represented. The monodromy matrix for the equation (1) is shown to give Lax representations for (3) and to generate a corresponding commom set of their involutive conservation laws.

The mentioned above objects are found in an exact form for the Manin-Radul supersymmetric hierarchy.