# ABSTRACT

# THE RIEMANN EXTENSIONS IN GEOMETRICAL THEORY OF DIFFERENTIAL EQUATIONS AND THEIR APPLICATIONS


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The properties of four and eight-dimensional Riemannian spaces $D^4$ and $D^8$ with the metrics

$$ds^2 = -2\Gamma^k_{ij}(x^l)\Psi_k dx^i dx^j + 2d\Psi_k dx^k \tag{1}$$

and the solutions of corresponding geodesics equations

$$\ddot{x}^k + \Gamma^k_{ij}\dot{x}^i\dot{x}^j = 0, \qquad \frac{\delta^2\Psi_k}{ds^2} + R^l_{kji}\dot{x}^j\dot{x}^i\Psi_l = 0, \tag{2}$$

are studied. Here $\Psi_k$ are the coordinates of additional space, $\frac{\delta\Psi_k}{ds} = \frac{d\Psi_k}{ds} - \Gamma^l_{jk}\Psi_l\frac{dx^j}{ds}$, $\Gamma^k_{ij} = \Gamma^k_{ij}(x^l)$ are the coefficients of affine connection of two and four-dimensional spaces and $R^l_{ijk}$ their curvature tensors.

The linear $4\times 4$ matrix system of second order ODE's for coordinates $\Psi_k$ in (2) has the form

$$\frac{d^2\vec{\Psi}}{ds^2} + A(s)\frac{d\vec{\Psi}}{ds} + B(s)\vec{\Psi} = 0$$

and their properties with the help of the invariants of the $4\times 4$ matrix-function $E = B - \frac{1}{2}\frac{dA}{ds} - \frac{1}{4}A^2$ are studied. The results for investigation of the properties of the geodesics of the spaces $D^2$ and $D^4$ are used. The applications at the theory of nonlinear dynamical systems in the case $D^2$ and for the Einstein spaces in $D^4$-case are considered.

The examples of translation surfaces $X^k(u,v)$ with the equations

$$\frac{\partial X^k}{\partial u\partial v} + \Pi^k_{ij}\frac{\partial X^i}{\partial u}\frac{\partial X^j}{\partial v} = 0$$

in the spaces with the metrics (1) are constructed and the properties of corresponding Beltrami-Laplace and Dirac operators of the metrics (1) are considered.

The methods of 3-dimensional Einstein-Weyl geometry for investigation of the initial value problem in theory of the second and third order ODE's are used.


## References


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