ON EQUIVARIANT BOUNDARY VALUE PROBLEMS

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Let Ω be an arbitrary bounded domain in the space $\mathbf{R}^n$ with the boundary $\partial\Omega$ and $\mathcal{L} = \sum_{|\alpha|\le m} a_\alpha(x) D^\alpha, D^\alpha = (-i\partial)^{|\alpha|}/\partial x_1^{\alpha_1}...\partial x_n^{\alpha_n}$, $\alpha \in \mathbf{Z}^n_+, |\alpha| = \sum_k \alpha_k$ be some formally self-ajoint differential operation with smooth complex matrix coefficients $a_\alpha(x)$, i.e. their elements belong to $C^\infty(\overline{\Omega})$. Let L_0 with domain $D(L_0)$ be a minimal operator and $L = (L_0)^*$ be a maximal operator of $\mathcal{L}$, $C(L) = D(L)/D(L_0)$ be a boundary space, $\Gamma : D(L) \to C(L)$ be a factor-mapping. An boundary value problem $Lu = f, \Gamma u \in B \subset C(L)$ is called well-posed if an expansion $L_B = L|_{D(L_B)}$, $D(L_B) = \Gamma^{-1}B$ has a continuous two-sided inverse operator.

Let G be some Lie group smoothly acting in the closed domain $\overline{\Omega}$, on boundary $\partial\Omega$ and the action of group remains volume of domain. Let differential operation $\mathcal{L}$ is invariant with respect to the group action, that is $g(\mathcal{L}u) = \mathcal{L}(gu)$. Then spaces $D(L)$, $D(L_0), C(L)$ are invariant with respect to the action of group G. Boundary value problem $Lu = f, \quad \Gamma u \in B$, let's name G-invariant, if space B is invariant with respect to the indicated action of group G. If the group G is compact, that, as it is well known, the Hilbert space of representation is decomposed in the direct sum of finite-dimensional invariant subspaces of irreducible representations. And if the group also is commutative, such representations are one-dimensional. Let space of a representation of the group G be the boundary space $C(L)$. For compact group we have decompositions

$C(L) = \sum_{k=0}^\infty \oplus \tilde{C}^k, \ C(\mathrm{ker}L) = \sum_{k=0}^\infty \oplus C^k(\mathrm{ker}L), \ B = \sum_{k=0}^\infty \oplus B^k.$

If ours G-invariant boundary value problem is well-posed, the decompositions in the direct sum $C(L) = C(\mathrm{ker}L) \oplus B$ attracts a decompositions in the direct sum $C^k := C^k(\mathrm{ker}L) \oplus B^k = \sum_l \tilde{C}^{k_l}$ with by finite-dimensional projectors $\Pi^k : C^k \to C^k(\mathrm{ker}L)$ along B^k and now a check of a well-posedness of G-invariant boundary boundary value problem can be shown by check of two properties:

1) $C^k(\mathrm{ker}L) \cap B^k = 0$; 2) $\exists \kappa > 0, \ \forall k, \ \|\Pi^k\|_{C^k} < \kappa$.

We investigate the spectrum of operator of a general well-posed SO-equivariant boundary value problem for the Poisson equation in a disk and in a ball, selecting cases of violation of well-posedness of the same problem for the Helmholz equation in violation of property 1). Thus the fulfilment of property 2) appears by the supplied property of well-posedness of the problem for the Poisson equation.

Refs: Burskii V.P. Investigation methods of boundary value problems for general differential equations. Kiev, Naukova dumka, 2002 (In Russian). e-mail:burskii@iamm.ac.donetsk.ua