MONOPOLE EQUATIONS ON 8-MANIFOLDS WITH SPIN(7) HOLONOMY: THE ENERGY INTEGRAL

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In Seiberg-Witten theory over a 4-manifold, a $U(1)$ connection is coupled to a spinor field and the Seiberg-Witten equations are the minimizers of an action involving the curvature 2-form F and a positive Dirac spinor ϕ^+. In the coupling of a Dirac spinor to the Yang-Mills action, the key is the Weitzenböck formula which relates the covariant derivatives $\nabla\phi^\pm$ to $D^\pm\phi^\pm$, where $D^\pm$ are Dirac operators, compensating for the scalar curvature term and bringing in the coupling $(\rho^\pm(F)\phi^\pm, \phi^\pm)$. The Seiberg-Witten equations relate $\rho^+(F^+)$ to the spinor field in such a way that the action is reduced to its topological lower bound [1].

In 8 dimensions, we work with a manifold M with Spin(7) holonomy. Such manifolds admit a globally defined, self-dual, closed 4-form Φ, called the Bonan form [2], which determine an orthogonal direct sum decomposition of the 2-forms on M as 7 and 21 dimensional eigenspaces of the map $\omega \to *(\Phi\omega)$, equivalent to the splitting in [3]. The appropriate set up for the coupling of a spinor field to the Yang-Mills action, is a spin^c strucure on M, leading to the spinor bundles $W^\pm$ together with the maps $\rho^\pm$, and to an associated $U(1)$ line bundle, called the virtual line bundle vith connection A and curvature F. The Weitzenböck formula relates the action of the Dirac operators on $W\pm$ to the L_2 norms of the the derivatives of the spinor field, to the scalar curvature s of M and to the curvature F of the virtual line bundle. For M, F and Φ as above, we define a projection operator on W^- via the Bonan 4-form as

$$P = \tfrac{1}{8} - \tfrac{1}{6}\rho^-(\Phi),$$

and define a section ϕ of W^- by

$$\phi = (1-P)U + JPU$$

where J is the complex structure on W^- and U is a section of W^- such that $(BU, U) = 0$ for every skew hermitian endomorphism B of W^-. Then the energy integral

$$\int_M \left[|F|^2 + |\nabla\phi|^2 + \tfrac{1}{4}s|\phi|^2 + |\phi\bar{\phi}^t - \bar{\phi}\phi^t|^2\right] dvol \geq \tfrac{2}{3}\int_M F\wedge F\wedge\Phi$$

reduces to its lower bound provided that the monopole equations

$$D^-\phi = 0, \quad \rho^-(F^{(7)}) = \tfrac{1}{2}(\phi\bar{\phi}^t - \bar{\phi}\phi^t), \quad \mathrm{div}A = 0$$

are satisfied [4].

Similar equations for the coupling to a positive spinor were given in [5]. Both systems of equations are inhomogeneous analogues of the elliptic system of equations given in Corrigan et.al [3] which also arise as minimizers of the L_2 norm of F [6].

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