НЕЛІНІЙНІ КОЛИВАННЯ т. 2 • № 4 • 1999

UDC 531 / 534 . 01

TIME, RELATIVITY AND PHYSICAL PRINCIPLE: GENERALIZATIONS AND APPLICATIONS

ЧАС, ВІДНОСНІСТЬ І ФІЗИЧНІ ПРИНЦИПИ: УЗАГАЛЬНЕННЯ І ЗАСТОСУВАННЯ

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A basis for a new relativity theory and a new physical concept are established. They lead to a further development of the theory of natural tracking control of nonlinear systems.

Встановлено концепцію нової теорії відносності та нових фізичних принципів. Це зумовлює подальший розвиток адекватної теорії керування нелінійними системами.

1. Introduction. Different physical sub-processes of a complex physical process can have different natural speeds of their evolutions. This inspired a study of processes and systems with multiple time scales [1 – 7]. Such examples are different speeds of fluidic, electrical, mechanical and thermal processes in chemical industry, process industry, and in power stations. Other examples are in control systems, in econometric systems, in biological systems. The fact that a change of a time scale influences the (numerical) value of the speed, and the fact that different natural velocities of evolutions of different processes can justify different time scales, have been well exploited to simplify the analysis of various dynamical system properties. The analysis is simplified by reducing their analysis to that of separated subprocesses or subsystems as well as to a study of their interactions, which appeared effective in particular in the framework of large scale dynamical systems [8].

The importance of a change of a time scale was well recognised and emphasised by Newton [9], who introduced and explained the meaning and defined the sense not only of the absolute time but also of the relative time that has been very rarely referred to in the literature on time.

The meaning and sense of time has been an intrinsic topic at least from Heraclit's epoch [10] and were considered from purely mathematical and systems point of view [11, 12], from physical points of view [10, 13 – 27] or from a philosophical standpoint [28 – 34]. Historical reviews can be found in [35, 36].

Using the Lorentz transformation of both time and space coordinates [37 – 41], which originated from a paper by Voigt as confirmed by Lorentz himself [40] and which was efficiently applied at the early development of the relativity theory by Poincaré [42], Einstein introduced and established another, different from Newton's, meaning and sense of the time relativity [43 – 47]. The modern literature on time has accepted Einstein's explication of time [10, 13 – 27] as the basic point of the relativity theory nevertheless whether it is treated purely mathematically

[41, 48 – 51] or also from a physical point of view [47, 48, 50, 52 – 62]. Another crucial point of the relativity theory is the postulate on the constancy of the velocity of light in vacuum relative to inertial frames, which has been interpreted as and used for invariance of its value relative to a change of a time scale [43 – 47, 63, 64].

Analysing all this the following questions have been risen:

Q_1. Is time a physical or purely mathematical variable, or even only a mathematical parameter?

Q_2. Is there an explanation and definition of time that can incorporate both Newton's and Einstein's?

Q_3. How does a change of a time scale influence the light speed value and should not be a time scale accepted before a light speed value is considered?

Q_4. What is an influence of a change of a time scale on Lorentz transformation via its influence on the light speed value?

Q_5. Is it possible to generalize the Lorentz transformation by obeying Einstein's conditions imposed on the transformation for its validity?

Since all these questions have been recently affirmatively replied by showing that the Lorentz transformation is just a singular case [65, 66], then they cause other questions to be posed, replied and explained:

Q_6. How the generalized Lorentz transformation from the references [65, 66] can be used further in physics, what are its implications on:

a) velocity;

b) acceleration;

c) mass;

d) force;

e) energy,

and whether essentially new relationships result?

Q_7. What are implications on the relativity theory, does the generalized Lorentz transformation lead to new views on the relativity theory, does it constitute a basis for a generalization of the relativity theory?

Q_8. Does the meaning, sense and definition of time accepted in [65, 66] provide another opening important for physics and possibly for other scientific and/or engineering theories and/or applications?

For the sake of the continuity and causality of the presentation and due to the delicacy and importance of replies to the posed questions the paper will present solutions to the first five questions by referring to [65, 66] where the proofs can be found. They enable us to reply to all other questions and to prove rigorously the statements, which is the primary goal of the paper. The results provide complete affirmative replies and explanations, and open various new avenues of research and development in physics, mathematics, systems and control (science and engineering). They also offer a new basis for philosophical understanding and interpretation of time. They also explain why every person has a feeling of her/his own time.

The accepted definition of time discovers no collision with Newton's meaning of the time relativity and simultaneously leads to new results in the relativity theory generalizing those by Lorentz [37 – 40] and Einstein [43 – 46], which therefore follow as special cases.

Furthermore, the properties of time clearly explained in and reflected by the accepted definition lead to a new physical principle to be stated, which appears in particular useful for development of novel directions in control science and engineering that will be also shown.

2. Physical variables and time. Accepted meanings of basic notions will be explained for the sake of preciseness and clearness of the presentation.

A **quantity** that can change its *value* is a *variable*. A variable is *a physical variable* if and only if there is a material object such that its (complete or partial) instantaneous internal (physical) situation at any its point is expressed by an instantaneous value of the variable at the same point and at the same moment. A physical variable can take any value from a *set of its values*, which is *infinite* and *everywhere dense*.

Time *is an independent physical variable the value of which is strictly monotonously continuously increasing independently of all other (physical and mathematical) variables, processes and events, which is used to uniquely determine the order of events happening*. The *value of time* is called **moment** or **instant** and is denoted by t or by τ and a subscript, e.g. t_2 or τ_b. An arbitrary instant will be denoted as time itself by t or by τ. The time value is determined accurately up to an unknown additive constant, that is that we do not know what is total zero time value: $t = 0$. We accept it conventionally like for some other variables (e.g. position, voltage).

Moment (instant) reflects *an instantaneous internal physical situation* of a material object called its **age**. **Time value difference** (**time interval**) is used to *measure duration of a process, of a movement or of a rest*.

Different materials can have *different speeds of ageing*, which can hold also for the same material object at its different points. For this reason, different time scales (e.g. decimal, logarithmic), different initial moments and / or different time units (e.g. second, minute) can be assigned to different material objects and / or to different parts of the same material object giving a relative meaning to time in this sense. This is *Newtonian meaning of the relativity of time.* It should be noted that Newton [9, p. 8 – 10] introduced and determined not only the absolute but also a relative sense of time. Einstein accepted another relative sense of time [43, p. 20; 45, p. 26 – 27; 46, p. 23 – 40].

With the above Newtonian relative sense of time in mind it is clear why different persons think that time has different speeds for them; i.e. that each of them has its own personal time. In fact, they think of the personal velocities of their processes. For example, if a discussion was interesting then its process went "fast" and a person usually say "time passed fast during the discussion", or if a discussion was uninteresting then the person would say "time passed slow", although in both cases the discussions could really last equally.

Since time value is strictly monotonously continuously increasing then $dt > 0$ is only meaningful. Time is indispensable for the definition and measurement of the speed (acceleration,...) of a value change of every variable. The speed value is measured relative to an accepted time scale. The former is possible only after having defined the latter, which means that we cannot determine a speed value if a time scale and unite have not been well defined.

A path (vector) $d\mathbf{r}$ passed by a light array over an infinitesimal time interval dt is the light speed (vector) $\mathbf{c}(t) = \dfrac{d\mathbf{r}(t)}{dt}$ at moment t. The value of the light speed $c(t) = \|\mathbf{c}(t)\|$ is constant in vacuum: $c(t) = c$ [43, p. 15; 45, p. 26], which is the light speed relative to the vacuum, for short: the light speed. In what follows, the environment will be arbitrary but fixed and such that it enables a constant light speed. The environment dimension $n \in \{1, 2, \ldots\}$. We consider it as an n-dimensional real vector space and denote it by R^n. A fixed point O can be accepted in the environment R^n to be the origin of a system of reference (a coordinate system). We can accept another point O_A movable relative to the environment to be the origin of another coordinate system denoted by R^n_A. The speed $\mathbf{v}_{0A}(t)$ of the point O_A and of the reference system

R_A^n relative to the environment R^n is accepted constant: $\mathbf{v}_{0A}(t) = \mathbf{v}_{0A} = \nu_{0A}\mathbf{r}_0$, where $\nu_{0A} = \|\mathbf{v}_{0A}\|$ is the Euclidean norm of $\mathbf{v}_{0A}$ and $\mathbf{r}_0$ is a unity vector, $\|\mathbf{r}_0\| = 1$.

A coordinate axis used for a time axis that is immovable relative to the environment is denoted by T. It is a set of all instants: $T = \{t: \ t \in R, \ t \in C^{(1)}(R), \ dt > 0\}$. Once a time axis has been accepted with a fixed time scale including a fixed time interval unit, then the zero time should have been also accepted. Any instant $\tau_0 \in T$ can be accepted for an initial instant. We accept that it has been chosen and fixed. It can be $\tau_0 = 0$.

The Cartesian product set $T \times R^n$ is denoted by J, $J = T \times R^n$. It is called the $(n+1)$-*dimensional real integral space*, for short *the integral space*. It enables a complete presentation of motions (of solutions of differential equations from a mathematical point of view, which are called integral trajectories). A pair (t, x) is called *an event in* J [11]. It can happen only once.

3. Physical continuity and uniqueness principle [67 – 70]. A physical variable value depends at least on both time and spatial coordinates of a point at which its value is considered. Such a dependence is *uniquely* determined. If the point is fixed then the physical variable can change its value only *continuously* in terms of time. These are real features of physical variables, which are expressed by the physical continuity and uniqueness principle.

Physical Continuity and Uniqueness Principle (PCUP):

a) *Physical Continuity Principle:*

A physical variable can change its value from one value to another one only by passing through all intermediate values.

b) *Physical Uniqueness Principle:*

A physical variable possesses a unique local instantaneous real value at any place (in any being or in any object) at any moment.

The continuity of time is crucial for applications of the Physical Continuity and Uniqueness Principle. It will be explained by analysing mathematical and physical responses of various nonlinear functions (nonlinearities) $y(\cdot): R \to R$, Fig. 1 through Fig. 5.

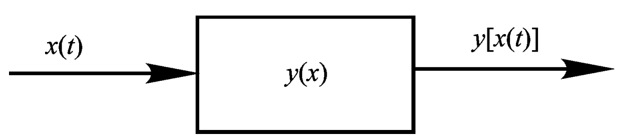

Fig. 1. A block representing a nonlinearity $y(\cdot)$.

The Physical Continuity and Uniqueness Principle enables us to explain how the Nature and beings exploit time continuity to create control. The Principle will be used in the sequel to explain and synthesise such a control of technical objects, which is called natural (tracking) control [71 – 80].

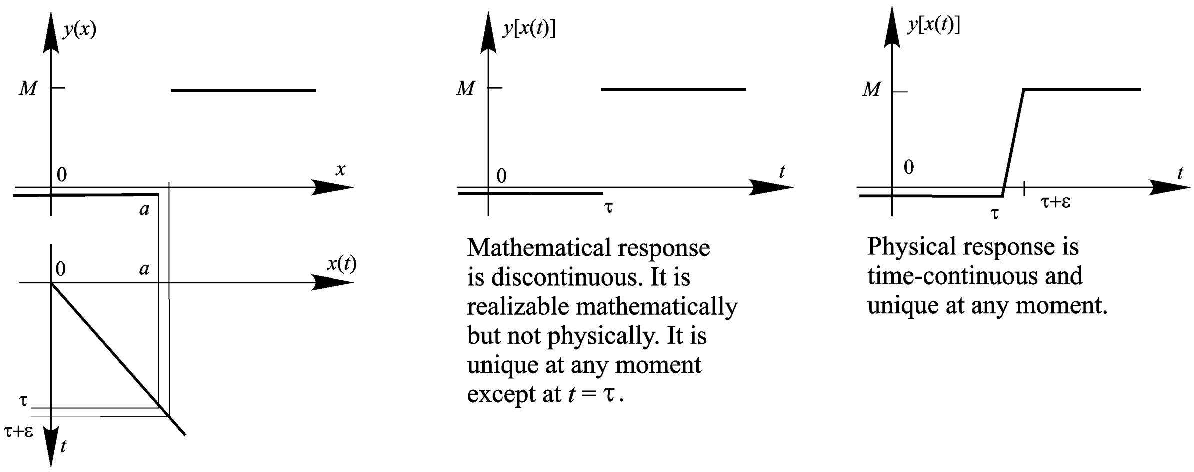

Fig. 2. Discontinuous nonlinearity $y(\cdot)$. It is double valued at $x = a$.

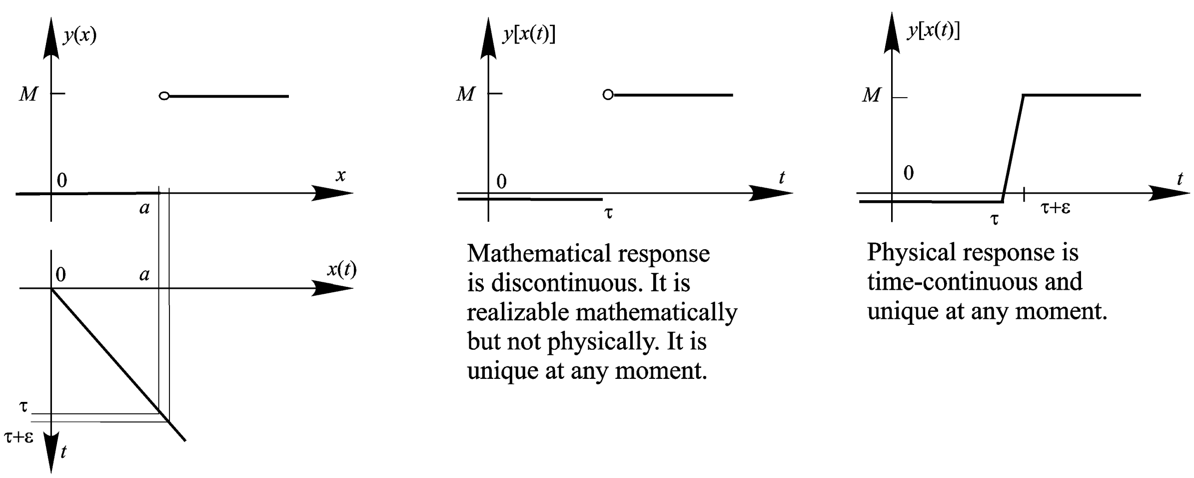

Fig. 3. Discontinuous nonlinearity $y(\cdot)$. It is single valued at $x = a$.

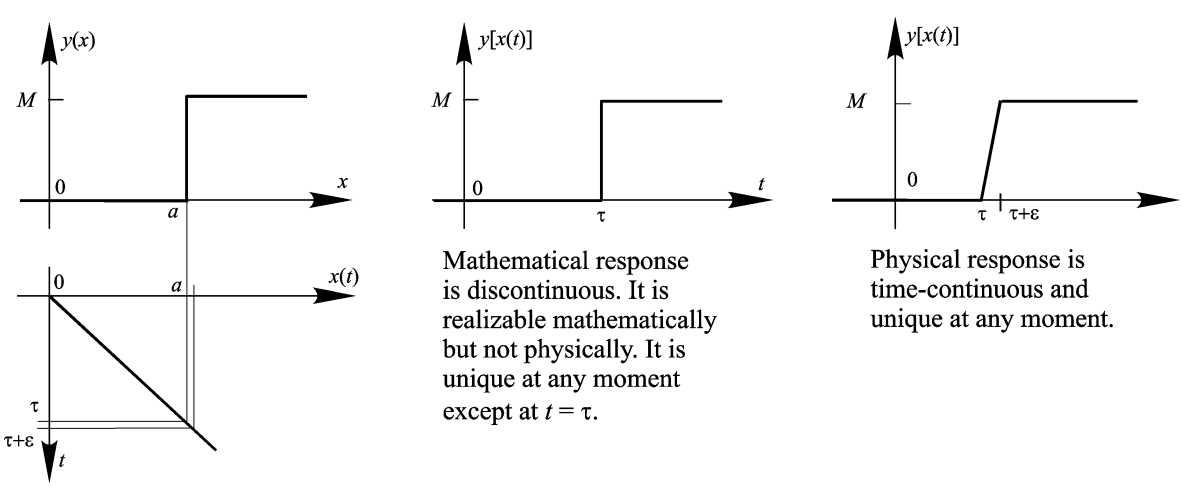

Fig. 4. Discontinuous nonlinearity $y(\cdot)$. It is multi-valued at $x = a$.

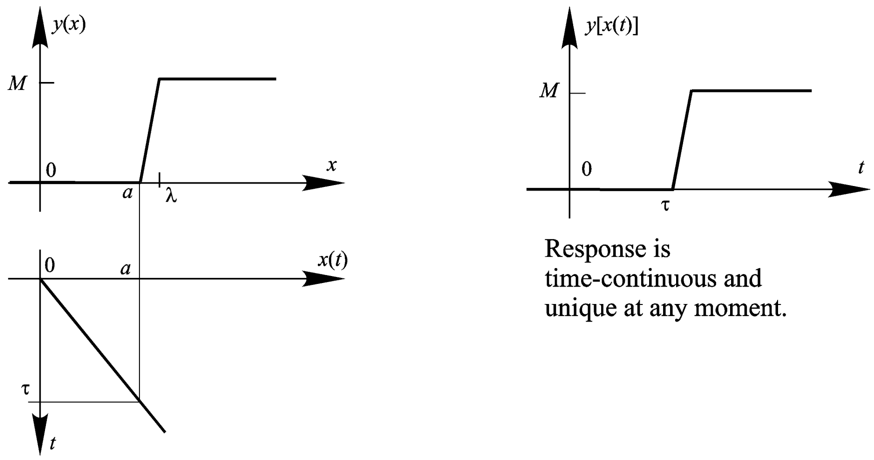

Fig. 5. Continuous nonlinearity $y(\cdot)$. It is single valued at $x = a$.

4. Time, speed and coordinate transformation [66]. What is presented in this section is from [66] (except the Corollary 1) and represents the basis for new results established in the next sections.

4.1. Time scale and speed. A time scale of a time axis T can be variously accepted. Let three different time scales be associated with T: "an original time scale" that is not indexed, "a"-time scale and "b"-time scale, which are then denoted respectively by T, T_a and T_b. A time value (instant) measured in T-scale, T_a-scale and T_b-scale is designated by τ, τ_a and τ_b, respectively, $\tau \in T$, $\tau_a \in T_a$ and $\tau_b \in T_b$. We accept $\tau_0 = \mu_a^{-1}\tau_{a0} = \mu_b^{-1}\tau_{b0} \in T$ and fix them in the sequel. Time scale factors μ_a and μ_b are positive numbers. Immovable time axes (such as T, T_a and T_b) are only considered in the what follows. Integral spaces corresponding to the time axes T, T_a and T_b are J, J_a and J_b, respectively:

$$J = T \times R\,, \quad J_a = T_a \times R_a^n \ \text{ and } \ J_b = T_b \times R_b^n\,.$$

Let the origin O_a of R_a^n and O_b of R_b^n move with constant $\mathbf{v}_{O_a}$ and $\mathbf{v}_{O_b}$ relative to the origin O of R^n. This means that the integral space J_a (J_b) is movable relative to the integral space J in the above sense that O_a and R_a^n (i.e. O_b and R_b^n) are only movable, but not the time axes T_a and T_b. Norms of the speeds $\mathbf{v}_{O_a}$ and $\mathbf{v}_{O_b}$ are denoted by v_{O_a} and v_{O_b}, when measured relative to $t \in T$. Since the light propagates equally in all directions then we can accept to consider a light signal and translations of O_a together with R_a^n and of O_b together with R_b^n in a direction and sense of an arbitrary unity vector $\mathbf{r}_0 \in R^n$ that is also used to represent symbolically the spaces R^n, R_a^n and R_b^n in Fig. 6.

Analogously, without losing in generality, we represent arbitrary elementwise constant vectors $\mathbf{r} \in R^n$, $\mathbf{r}_a \in R_a^n$ and $\mathbf{r}_b \in R_b^n$ as $\mathbf{r} = \rho\mathbf{r}_0$, $\mathbf{r}_a = \rho_a\mathbf{r}_0$ and $\mathbf{r}_b = \rho_b\mathbf{r}_0$. Their lengths expressed by their norms $\rho = \|\mathbf{r}\|$, $\rho_a = \|\mathbf{r}_a\|$ and $\rho_b = \|\mathbf{r}_b\|$, respectively, do not depend on a time scale used. Velocities should be naturally permitted to depend on a time scale used:

$\mathbf{v}_{O_a}(\tau_a) \equiv \mathbf{v}_{O_a}^a \equiv v_{O_a}^a\mathbf{r}_0$ — speed of O_a relative to O at an instant τ_a measured in terms of τ_a, $\mathrm{v}_{O_a}^a \le \mathrm{v}_{O_b}^a$ is accepted,

$\mathbf{v}_{O_b}(\tau_b) \equiv \mathbf{v}_{O_b}^b \equiv v_{O_b}^b\mathbf{r}_0$ — speed of O_b relative to O at an instant τ_b measured in terms of τ_b, $v_{O_a}^b \le v_{O_b}^b$ is accepted,

$\boldsymbol{\nu}(\tau_a) \equiv \boldsymbol{\nu}^a \equiv \nu^a\mathbf{r}_0 \equiv (v_{O_b}^a - v_{O_a}^a)\mathbf{r}_0$ — relative speed of O_b with respect to O_a at instant τ_a measured all in terms of τ_a; it is to be noted that $\nu^a \neq -c^a$ due to the accepted $v_{O_a}^a \le v_{O_b}^a$,

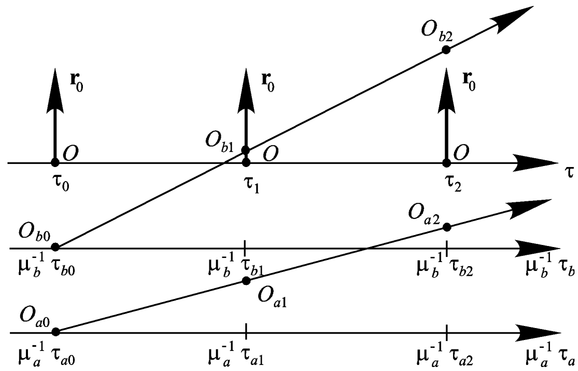

Fig. 6. Movable origins O_a of R_a^n and O_b of R_b^n, as well as R_a^n and R_b^n themselves relative to the origin O of R^n and relative to R^n itself. The time axes T, T_a and T_b are immovable.

c — the value of the light speed measured relative to $\tau \in T$,

c^a — the value of the light speed measured relative to $\tau_a \in T_a$,

$\boldsymbol{\nu}(\tau_b) \equiv\ \boldsymbol{\nu}^b \equiv \nu^b \mathbf{r}_0 \equiv (v^b_{O_b} - v^b_{O_a})\mathbf{r}_0$ — relative speed of O_b with respect to O_a at instant τ_b measured all in terms of τ_b; it is accepted that $\nu^b \neq c^b$,

c^b — the value of the light speed measured relative to $\tau_b \in T_b$.

4.2. Problem statement. What are values of the coefficients k_a^b and k_b^a, α_a^b and α_b^a such that the equations (1),

$$\mathbf{r}_a(\tau_a;\tau_{a0}) = k_b^a \left[\mathbf{r}_b(\tau_b;\tau_{b0}) - \nu^b(\tau_b - \tau_{b0})\mathbf{r}_0\right] , \tag{1a}$$

$$\mathbf{r}_b(\tau_b;\tau_{b0}) = k_a^b \left[\mathbf{r}_a(\tau_a;\tau_{a0}) + \nu^a(\tau_a - \tau_{a0})\mathbf{r}_0\right] , \tag{1b}$$

and the equations (2),

$$\tau_a - \tau_{a0} = \alpha_b^a \left[(\tau_b - \tau_{b0}) - \frac{\nu^b}{(c^b)^2}\,\rho_b(\tau_b;\tau_{b0})\right] , \tag{2a}$$

$$\tau_b - \tau_{b0} = \alpha_a^b \left[(\tau_a - \tau_{a0}) + \frac{\nu^a}{(c^a)^2}\,\rho_a(\tau_a;\tau_{a0})\right] , \tag{2b}$$

hold and that they imply both the equations (3),

$$\tau_a - \tau_{a0} = \mu_a(t - t_0)\,, \quad \mu_a \in R^+ =]0,\infty[\,, \tag{3a}$$

$$\tau_b - \tau_{b0} = \mu_b(t - t_0)\,, \quad \mu_b \in R^+\,, \tag{3b}$$

and the equation (4),

$$r_a^2(\tau_a;\tau_{a0}) - \left[c^a(\tau_a - \tau_{a0})\right]^2 = r_b^2(\tau_b;\tau_{b0}) - \left[c^b(\tau_b - \tau_{b0})\right]^2 \tag{4}$$

and what are values of the factors μ_a and μ_b?

The time-scale equations (3) are inherent for multiple time scale systems. The condition (4) is a crucial condition of the Einstein relativity theory for validity of the coordinate transformation defined by (1) and (2) [43, 44].

4.3. Problem solution. Let a light signal be emitted from $O = O_{a0} = O_{b0}$ at an initial instant $\tau_0 = \mu_a^{-1}\tau_{a0} = \mu_b^{-1}\tau_{b0}$. Let its position be determined by $\mathbf{r}(\tau;\tau_0)$ in R^n at an instant τ; by $\mathbf{r}_a(\tau_a;\tau_{a0})$ in R_a^n at τ_a and by $\mathbf{r}_b(\tau_b;\tau_{b0})$ in R_b^n at τ_b. Hence:

$$\mathbf{r}\left(\tau;\tau_0\right) = c\left(\tau-\tau_0\right)\mathbf{r}_0 = \mathbf{r}\left(\tau-\tau_0;0\right), \tag{5a}$$

$$\mathbf{r}_a\left(\tau_a;\tau_{a0}\right) = c^a\left(\tau_a-\tau_{a0}\right)\mathbf{r}_v = \mathbf{r}_a\left(\tau_a-\tau_{a0};0\right), \tag{5b}$$

and

$$\mathbf{r}_b\left(\tau_b;\tau_{b0}\right) = c^b\left(\tau_b-\tau_{b0}\right)\mathbf{r}_0 = \mathbf{r}_b\left(\tau_b-\tau_{b0};0\right). \tag{5c}$$

The distance vectors can be expressed also in terms of their lengths (norms),

$$\mathbf{r}\left(\tau;\tau_0\right) = \rho\left(\tau-\tau_0\right)\mathbf{r}_0, \tag{6a}$$

$$\mathbf{r}_a\left(\tau_a;\tau_{a0}\right) = \rho_a\left(\tau_a-\tau_{a0}\right)\mathbf{r}_0, \tag{6b}$$

and

$$\mathbf{r}_b\left(\tau_b;\tau_{b0}\right) = \rho_b\left(\tau_b-\tau_{b0}\right)\mathbf{r}_0. \tag{6c}$$

Two cases should be distinguished and will be considered:

General case: $k_a^b \neq k_b^a$ and / or $\alpha_a^b \neq \alpha_b^a$. All the coefficients k_a^b, k_b^a, α_a^b and $\alpha_b^a \in R^+ =]0,\infty[$.

Special case: $k_a^b = k_b^a = k \in R^+$ and $\alpha_a^b = \alpha_b^a = \alpha \in R^+$.

Problem solution for the general case.

Theorem 1. *In order for the coefficients $k_a^b \in R^+$, $k_b^a \in R^+$, $k_a^b \neq k_b^a$, $\alpha_a^b \in R^+$ and $\alpha_b^a \in R^+$, $\alpha_a^b \neq \alpha_b^a$, to obey (1) and (2), and for (1) and (2) to imply (3) and (4) it is necessary and sufficient that the following equations hold for any choice of $\mu_a \in R^+$:*

$$\alpha_b^a = \frac{1}{1-\dfrac{\nu^b}{c^b}}\,\frac{\mu_a}{\mu_b} = \frac{c^b}{c^a}\,k_b^a, \tag{7a}$$

$$\alpha_a^b = \frac{1}{1+\dfrac{\nu^a}{c^a}}\,\frac{\mu_b}{\mu_a} = \frac{c^a}{c^b}\,k_a^b, \tag{7b}$$

$$\nu^b < c^b, \tag{7c}$$

$$k_a^b = \frac{1}{1+\dfrac{\nu^a}{c^a}}, \tag{8a}$$

$$k_b^a = \frac{1}{1-\dfrac{\nu^b}{c^b}}, \tag{8b}$$

and

$$\mu_b = \frac{c^a}{c^b}\,\mu_a\,. \tag{9}$$

Proof. It is given in Appendix A of [66].

The equations (7) and (8) give the next form to the equations (1) and (2):

$$\mathbf{r}_a(\tau_a;\tau_{a0}) = \frac{\left[\mathbf{r}_b(\tau_b;\tau_{b0}) - \nu^b(\tau_b - \tau_{b0})\mathbf{r}_0\right]}{1 - \dfrac{\nu^b}{c^b}}\,, \tag{1a$'$}$$

$$\mathbf{r}_b(\tau_b;\tau_{b0}) = \frac{\left[\mathbf{r}_a(\tau_a;\tau_{a0}) - \nu^a(\tau_a - \tau_{a0})\mathbf{r}_0\right]}{1 + \dfrac{\nu^a}{c^a}}\,, \tag{1b$'$}$$

$$\tau_a - \tau_{a0} = \frac{c^b}{c^a}\,\frac{1}{1 - \dfrac{\nu^b}{c^b}}\,\left[(\tau_b - \tau_{b0}) - \frac{\nu^b}{(c^b)^2}\,\rho_b(\tau_b;\tau_{b0})\right]\,, \tag{2a$'$}$$

$$\tau_b - \tau_{b0} = \frac{c^a}{c^b}\,\frac{1}{1 + \dfrac{\nu^a}{c^a}}\,\left[(\tau_a - \tau_{a0}) + \frac{\nu^a}{(c^a)^2}\,\rho_a(\tau_a;\tau_{a0})\right]\,. \tag{2b$'$}$$

The definition of the speed, and the equations (3) and (9) verify directly the following known relationship among time scales and velocity scales, which are linked with the corresponding light speed values.

Corollary 1. *Let* $\mathbf{v}^a = v^a\mathbf{r}_0 = v(\tau_a;\tau_{a0})\,\mathbf{r}_0$ *and* $\mathbf{v}^b = v^b\mathbf{r}_0 = v(\tau_b;\tau_{b0})\,\mathbf{r}_0$ *be the velocities of the same point in the same space coordinate system* R^n *but relative to time measured in two different time scales* T_a *and* T_b*. Let the corresponding values of the light speed be* $\mathbf{c}^a = c^a\mathbf{r}_0 = c(\tau_a;\tau_{a0})\mathbf{r}_0$ *and* $\mathbf{c}^b = c^b\mathbf{r}_0 = c(\tau_b;\tau_{b0})\mathbf{r}_0$*, respectively. Then:*

$$\frac{v^a}{v^b} = \frac{\mu_b}{\mu_a} = \frac{c^a}{c^b}\,.$$

In the case the coordinate systems R^n_a and R^n_b move with the same velocities then the next theorem is easily proved by setting $\nu^a = \nu^b = 0$ in the proof of Theorem 1 (Appendix A in [66]):

Theorem 2. *Let the coordinate systems* R^n_a *and* R^n_b *move with the same velocities. In order for the coefficients* $k^b_a \in R^+$, $k^a_b \in R^+$, $\alpha^b_a \in R^+$ *and* $\alpha^a_b \in R^+$, $\alpha^b_a \neq \alpha^a_b$*, to obey (1) and (2), and for (1) and (2) to imply (3) and (4) it is necessary and sufficient that the following equations hold for any choice of* $\mu_a \in R^+$*:*

$$\alpha^a_b = \frac{\mu_a}{\mu_b} = \frac{c^b}{c^a}\,, \quad \alpha^b_a = \frac{\mu_b}{\mu_a} = \frac{c^a}{c^b} = (\alpha^a_b)^{-1}\,, \quad \textit{and} \quad k^b_a = k^a_b = 1\,.$$

This result shows that different time scales can be chosen in the case $\alpha^b_a \neq \alpha^a_b$ even the coordinate systems R^n_a and R^n_b move with the same velocities. This is a consequence of the time independence of the space. Then the time scale coefficients are related to the corresponding values of the light speed by the equation (9). Hence, it is still possible to model dynamic systems with multiple time scales by applying the coordinate transformation defined by (1) and (2) or equivalently by $(1')$ and $(2')$.

Problem solution for the special case.

Theorem 3. *In order for the coefficients $k_a^b \in R^+$, $k_b^a \in R^+$, $k_a^b = k_b^a = k$, $\alpha_a^b \in R^+$ and $\alpha_b^a \in R^+$, $\alpha_a^b = \alpha_b^a = \alpha$, to obey (1) and (2), and for (1) and (2) to imply (3) and (4) it is necessary and sufficient that the following equations hold for any choice of $\mu_a \in R^+$:*

$$c^a = c^b = c\,, \tag{10}$$

$$\nu^a = \nu^b = \nu\,, \tag{11}$$

$$\alpha = k = \frac{1}{\sqrt{1 - \dfrac{\nu^2}{c^2}}}\,, \tag{12}$$

$$\mu_b = \mu_a \sqrt{\frac{1 + \dfrac{\nu}{c}}{1 - \dfrac{\nu}{c}}}\,. \tag{13}$$

Proof. It is shown in Appendix B of [66].

The preceding theorem gives the following special form to the equations (1):

$$\mathbf{r}\,(\tau_a; \tau_{a0}) = \frac{[\mathbf{r}(\tau_b; \tau_{b0}) - \nu(\tau_b - \tau_{b0})\,\mathbf{r}_0]}{\sqrt{1 - \dfrac{\nu^2}{c^2}}}\,, \tag{14a}$$

$$\mathbf{r}\,(\tau_b; \tau_{b0}) = \frac{[\mathbf{r}(\tau_a; \tau_{a0}) + \nu(\tau_a - \tau_{a0})\,\mathbf{r}_0]}{\sqrt{1 - \dfrac{\nu^2}{c^2}}}\,, \tag{14b}$$

and to the equations (2),

$$\tau_a - \tau_{a0} = \frac{(\tau_b - \tau_{b0}) - \dfrac{\nu}{c^2}\,\rho_b(\tau_b; \tau_{b0})}{\sqrt{1 - \dfrac{\nu^2}{c^2}}}\,, \tag{15a}$$

$$\tau_b - \tau_{b0} = \frac{(\tau_a - \tau_{a0}) + \dfrac{\nu}{c^2}\,\rho_a(\tau_a; \tau_{a0})}{\sqrt{1 - \dfrac{\nu^2}{c^2}}}\,. \tag{15b}$$

These equations are well known as the equations of the Lorentz transformation [37 – 41, 51]. They are basic for the Einstein theory of relativity [43 – 46]. It is now clear that they appear a singular case of the transformations (1) and (2). It should be also noted that they follow herein from the accepted time definition in Newton's sense.

If the reference systems R_a^n and R_b^n move with the same speed then the preceding theorem reduces to:

Corollary 2. *Let the coordinate systems R_a^n and R_b^n move with the same velocities. In order for the coefficients $k_a^b \in R^+$, $k_b^a \in R^+$, $k_a^b = k_b^a = k$, $\alpha_a^b \in R^+$ and $\alpha_b^a \in R^+$, $\alpha_a^b = \alpha_b^a = \alpha$, to obey (1) and (2), and for (1) and (2) to imply (3) and (4) it is necessary and sufficient that the following equations hold for any choice of $\mu_a \in R^+$:*

$$c^a = c^b = c\,, \quad \alpha = k = 1\,, \quad \mu_b = \mu_a\,. \tag{16}$$

An illustrative example is worked out in [66].

5. Dynamical variables and coordinate transformation. In this and next sections new relationships will be derived by employing the generalized Lorentz transformation $(1')$, $(2')$ and the results obtained in [65, 66] that are presented in the preceding section.

5.1. Speed and coordinate transformation. *The general case.*

Theorem 4. *Let the coefficients $k_a^b \in R^+$, $k_b^a \in R^+$, $k_a^b \neq k_b^a$, $\alpha_a^b \in R^+$ and $\alpha_b^a \in R^+$, $\alpha_a^b \neq \alpha_b^a$, obey (1) and (2), and let (1) and (2) imply (3) and (4). Then the speed $\mathbf{v}_A^{O_A}(\tau_a;\tau_{a0})$ of a point A relative to the origin O_A of R_a expressed as a function of τ_a and the speed $\mathbf{v}_A^{O_B}(\tau_b;\tau_{b0})$ of the point A relative to the origin O_B of R_b expressed as a function of τ_b are interrelated as follows:*

$$\mathbf{v}_A^{O_A}(\tau_a;\tau_{a0}) = \frac{c^a}{c^b}\,\frac{\upsilon_A^{O_B}(\tau_b;\tau_{b0}) - \nu^b}{1 - \dfrac{\nu^b \upsilon_A^{O_B}(\tau_b;\tau_{b0})}{(c^b)^2}}\,\mathbf{r}_0\,, \tag{17a}$$

and

$$\mathbf{v}_A^{O_B}(\tau_b;\tau_{b0}) = \frac{c^b}{c^a}\,\frac{\upsilon_A^{O_A}(\tau_a;\tau_{a0}) + \nu^a}{1 + \dfrac{\nu^a \upsilon_A^{O_A}(\tau_a;\tau_{a0})}{(c^a)^2}}\,\mathbf{r}_0\,. \tag{17b}$$

Proof. Let all the conditions of the theorem statement hold. Then, the equations $(1')$ and $(2')$ also hold (Theorem 1). The speed $\mathbf{v}_A^{O_A}(t_a)$ is defined by:

$$\mathbf{v}_A^{O_A}(\tau_a;\tau_{a0}) = \frac{d\mathbf{r}_A^{O_A}(\tau_a;\tau_{a0})}{d\tau_a}\,.$$

This equation, and the equations $(1')$ and $(2')$ now yield:

$$\mathbf{v}_A^{O_A}(\tau_a;\tau_{a0}) = \frac{d\mathbf{r}_A^{O_A}(\tau_a;\tau_{a0})}{d\tau_a} = $$

$$= \frac{\dfrac{d[\mathbf{r}_A^{O_B}(\tau_b;\tau_{b0}) - \nu^b(\tau_b - \tau_{bo})\mathbf{r}_0]}{1-\nu^b/c^b}}{d\left\{\dfrac{c^b}{c^a}\,\dfrac{1}{1-\nu^b/c^b}\left[(\tau_b - \tau_{b0}) - \dfrac{\nu^b}{(c^b)^2}\rho_A^{O_B}(\tau_b;\tau_{b0})\right]\right\}} = \frac{c^a}{c^b}\,\frac{\mathbf{v}_A^{O_B}(\tau_b;\tau_{b0}) - \nu^b\mathbf{r}_0}{1 - \dfrac{\nu^b}{(c^b)^2}\upsilon_A^{O_B}(\tau_b;\tau_{b0})}\,,$$

which proves (17a). The equation (17b) is analogously proved by starting with the definition of $\mathbf{v}_A^{O_B}(\tau_b;\tau_{b0})$,

$$\mathbf{v}_A^{O_B}(\tau_b;\tau_{b0}) = \frac{d\mathbf{r}_A^{O_B}(\tau_b;\tau_{b0})}{d\tau_b}\,, \tag{18}$$

and by using $(1\mathrm{b}')$ and $(2\mathrm{b}')$.

The velocity transformation equations (17) are beyond those of the relativity theory. The former are general, while the latter result from the special case. Notice once more that $\nu^a = = \nu^b = \nu$ and $c^a = c^b = c$ in the special case.

The special case.

Theorem 5. *Let the coefficients $k_a^b \in R^+$, $k_b^a \in R^+$, $k_a^b = k_b^a = k$, $\alpha_a^b \in R^+$ and $\alpha_b^a \in R^+$, $\alpha_a^b = \alpha_b^a = \alpha$, obey (1) and (2), and let (1) and (2) imply (3) and (4). Then the speed $\mathbf{v}_A^{O_A}(\tau_a;\tau_{a0})$ of a point A relative to the origin O_A of R_a expressed as a function of τ_a and the speed $\mathbf{v}_A^{O_B}(\tau_b;\tau_{b0})$ of the point A relative to the origin O_B of R_b expressed as a function of τ_b are interrelated as follows:*

$$\mathbf{v}_A^{O_A}(\tau_a;\tau_{a0}) = \frac{v_A^{O_B}(\tau_b;\tau_{b0}) - \nu}{1 - \dfrac{\nu v_A^{O_B}(\tau_b;\tau_{b0})}{c^2}}\,\mathbf{r}_0\,, \tag{19a}$$

and

$$\mathbf{v}_A^{O_B}(\tau_b;\tau_{b0}) = \frac{v_A^{O_A}(\tau_a;\tau_{a0}) + \nu}{1 + \dfrac{\nu v_A^{O_A}(\tau_a;\tau_{a0})}{c^2}}\,\mathbf{r}_0\,. \tag{19b}$$

Proof. Let all the conditions of the theorem statement hold. Then, the equations (14) and (15) also hold (Theorem 3). The speed $\mathbf{v}_A^{O_A}$ is determined by the equations (14a) and (15a) as follows:

$$\mathbf{v}_A^{O_A}(\tau_a;\tau_{a0})\frac{d\mathbf{r}_A^{O_A}(\tau_a;\tau_{a0})}{d\tau_a} =$$

$$= \frac{\dfrac{d[\mathbf{r}_A^{O_B}(\tau_b;\tau_{b0}) - \nu(\tau_b - \tau_{bo})\mathbf{r}_0]}{\sqrt{1-\nu^2/c^2}}}{\dfrac{d\left[(\tau_b - \tau_{b0}) - \dfrac{\nu}{c^2}\rho_b(\tau_b;\tau_{b0})\right]}{\sqrt{1-\nu^2/c^2}}} : \frac{d\tau_b}{d\tau_b} = \frac{\mathbf{v}_A^{O_B}(\tau_b;\tau_{b0}) - \nu\mathbf{r}_0}{1 - \dfrac{\nu}{c^2}v_A^{O_B}(\tau_b;\tau_{b0})}\,,$$

which implies (19a). The equation (19b) is analogously proved by starting with the definition of $\mathbf{v}_A^{O_B}(\tau_b;\tau_{b0})$, (18), and by using (14b) and (15b).

Notice that the equations (19) can be also derived directly from the equations (17) for $c^a = = c^b = c$ due to Corollary 1 and the conditions of Theorem 5.

The equations (19) are well known in the relativity theory. It is important to note that they are derived herein starting with the time definition in Newton's sense rather than in Einstein's.

5.2. Acceleration and coordinate transformation. *The general case.*

Theorem 6. *Let the coefficients $k_a^b \in R^+$, $k_b^a \in R^+$, $k_a^b \neq k_b^a$, $\alpha_a^b \in R^+$ and $\alpha_b^a \in R^+$, $\alpha_a^b \neq \alpha_b^a$, obey (1) and (2), and let (1) and (2) imply (3) and (4). Then the acceleration $\mathbf{a}_A^{O_A}(\tau_a;\tau_{a0})$ of a point A relative to the origin O_A of R_a expressed as a function of τ_a and the acceleration $\mathbf{a}_A^{O_B}(\tau_b;\tau_{b0})$ of the point A relative to the origin O_B of R_b expressed as a function of τ_b are interrelated as follows:*

$$\mathbf{a}_A^{O_A}(\tau_a;\tau_{a0}) = \left(\frac{c^a}{c^b}\right)^2\left(1+\frac{\nu^b}{c^b}\right)\frac{\left(1-\dfrac{\nu^b}{c^b}\right)^2}{\left[1 - \dfrac{\nu^b v_A^{O_B}(\tau_b;\tau_{b0})}{(c^b)^2}\right]^3}\,\mathbf{a}_A^{O_B}(\tau_b;\tau_{b0})\,, \tag{20a}$$

and

$$\mathbf{a}_A^{O_B}(\tau_b;\tau_{b0}) = \left(\frac{c^b}{c^a}\right)^2 \left(1-\frac{\nu^a}{c^a}\right) \frac{\left(1+\dfrac{\nu^a}{c^a}\right)^2}{\left[1+\dfrac{\nu^a v_A^{O_B}(\tau_a;\tau_{a0})}{(c^b)^2}\right]^3} \mathbf{a}_A^{O_A}(\tau_a;\tau_{a0})\,. \tag{20b}$$

Proof. Let all the conditions of the theorem statement hold. Then, the equations $(2')$ and (17) also hold (Theorem 1 and Theorem 4). The acceleration $\mathbf{a}_A^{O_A}(\tau_a;\tau_{a0})$ is defined by:

$$\mathbf{a}_A^{O_A}(\tau_a;\tau_{a0}) = \frac{d\mathbf{v}_A^{O_A}(\tau_a;\tau_{a0})}{d\tau_a}\,.$$

This equation, the equations (17a) and (2a′) now yield:

$$\mathbf{a}_A^{O_A}(\tau_a;\tau_{a0}) =$$

$$= \frac{d\left[\dfrac{c^a}{c^b}\,\dfrac{\mathbf{v}_A^{O_B}(\tau_b;\tau_{b0})-\nu^b\mathbf{r}_0}{1-\dfrac{\nu^b\mathbf{v}_A^{O_B}(\tau_b;\tau_{b0})}{(c^b)^2}}\right]}{d\left\{\dfrac{c^b}{c^a}\,\dfrac{1}{1-\dfrac{\nu^b}{c^b}}\left[(\tau_b-\tau_{b0})-\dfrac{\nu^b}{(c^b)^2}\rho_A^{O_B}(\tau_b;\tau_{b0})\right]\right\}} : \frac{d\tau_b}{d\tau_b} = \left(1-\frac{\nu^b}{c^b}\right)\left(\frac{c^a}{c^b}\right)^2 \times$$

$$\times \frac{\left[\dfrac{dv_A^{O_B}(\tau_b;\tau_{b0})}{d\tau_b}\right]\left[1-\dfrac{\nu^b v_A^{O_B}(\tau_b;\tau_{b0})}{(c^b)^2}\right]-\left[v_A^{O_B}(\tau_b;\tau_{b0})-\nu^b\right]\left[-\dfrac{\nu^b}{(c^b)^2}\,\dfrac{dv_A^{O_B}(\tau_b;\tau_{b0})}{d\tau_b}\right]}{\left[1-\dfrac{\nu^b\mathbf{v}_A^{O_B}(\tau_b;\tau_{b0})}{(c^b)^2}\right]^2\left\{\left[1-\dfrac{\nu^b}{(c^b)^2}\,\dfrac{d\rho_A^{O_B}(\tau_b;\tau_{b0})}{d\tau_b}\right]\right\}}\mathbf{r}_0 =$$

$$= \left(\frac{c^a}{c^b}\right)^2\left(1-\frac{\nu^b}{c^b}\right)\frac{1-\left(\dfrac{\nu^b}{c^b}\right)^2}{\left[1-\dfrac{\nu^b v_A^{O_B}(\tau_b;\tau_{b0})}{(c^b)^2}\right]^3}\mathbf{a}_A^{O_B}(\tau_b;\tau_{b0}) =$$

$$= \left(\frac{c^a}{c^b}\right)^2\left(1+\frac{\nu^b}{c^b}\right)\frac{\left(1-\dfrac{\nu^b}{c^b}\right)^2}{\left[1-\dfrac{\nu^b v_A^{O_B}(\tau_b;\tau_{b0})}{(c^b)^2}\right]^3}\mathbf{a}_A^{O_B}(\tau_b;\tau_{b0}),$$

which proves (20a). The equation (20b) is analogously proved by starting with the definition of $\mathbf{a}_A^{O_B}(\tau_b;\tau_{b0})$,

$$\mathbf{a}_A^{O_B}(\tau_b;\tau_{b0}) = \frac{d\mathbf{v}_A^{O_B}(\tau_b;\tau_{b0})}{d\tau_b}, \tag{21}$$

and by using (17b) and (2b′).

The acceleration transformation equations (20) are beyond those of the relativity theory, The former are general, while the latter result from the special case as shown in what follows.

The special case.

Theorem 7. *Let the coefficients $k_a^b \in R^+$, $k_b^a \in R^+$, $k_a^b = k_b^a = k$, $\alpha_a^b \in R^+$ and $\alpha_b^a \in R^+$, $\alpha_a^b = \alpha_b^a = \alpha$, obey (1) and (2), and let (1) and (2) imply (3) and (4). Then the acceleration $\mathbf{a}_A^{O_A}(\tau_a;\tau_{a0})$ of a point A relative to the origin O_A of R_a expressed as a function of τ_a and the acceleration $\mathbf{a}_A^{O_B}(\tau_b;\tau_{b0})$ of the point A relative to the origin O_B of R_b expressed as a function of τ_b are interrelated as follows:*

$$\mathbf{a}_A^{O_A}(\tau_a;\tau_{a0}) = \frac{\left[1-\left(\dfrac{\nu}{c}\right)^2\right]^{3/2}\mathbf{a}_A^{O_B}(\tau_b;\tau_{b0})}{\left[1-\dfrac{\nu v_A^{O_B}(\tau_b;\tau_{b0})}{c^2}\right]^3}, \tag{22a}$$

and

$$\mathbf{a}_A^{O_B}(\tau_b;\tau_{b0}) = \frac{\left[1-\left(\dfrac{\nu}{c}\right)^2\right]^{3/2}\mathbf{a}_A^{O_A}(\tau_a;\tau_{a0})}{\left[1-\dfrac{\nu v_A^{O_A}(\tau_a;\tau_{a0})}{c^2}\right]^3}. \tag{22b}$$

Proof. Let all the conditions of the theorem statement hold. Then, the equations (19) also hold (Theorem 5). The acceleration $\mathbf{a}_A^{O_A}(\tau_a;\tau_{a0})$ is determined by the equations (19a) and (15a) as follows:

$$\mathbf{a}_A^{O_A}(\tau_a;\tau_{a0}) = \frac{d\mathbf{v}_A^{O_A}(\tau_a;\tau_{a0})}{d\tau_a} =$$

$$= \left\{d\left[\frac{\mathbf{v}_A^{O_B}(\tau_b;\tau_{b0})-\nu\mathbf{r}_0}{1-\dfrac{\nu v_A^{O_B}(\tau_b;\tau_{b0})}{c^2}}\right]\right\}\left\{\frac{d\left[(\tau_b-\tau_{b0})-\dfrac{\nu\rho_A^{O_B}(\tau_b;\tau_{b0})}{c^2}\right]}{\sqrt{1-\dfrac{\nu^2}{c^2}}}\right\}^{-1} : \frac{d\tau_b}{d\tau_b} =$$

$$= \sqrt{1-\frac{\nu^2}{c^2}}\,\frac{\mathbf{a}_A^{O_B}(\tau_b;\tau_{b0}) - \mathbf{a}_A^{O_B}(\tau_b;\tau_{b0})\dfrac{\nu v_A^{O_B}(\tau_b;\tau_{b0})}{c^2} - \left[\mathbf{v}_A^{O_B}(\tau_b;\tau_{b0})-\nu\mathbf{r}_0\right]\left[-\dfrac{\nu\mathbf{a}_A^{O_B}(\tau_b;\tau_{b0})}{c^2}\right]}{\left[1-\dfrac{\nu v_A^{O_B}(\tau_b;\tau_{b0})}{c^2}\right]^2\left[1-\dfrac{\nu v_A^{O_B}(\tau_b;\tau_{b0})}{c^2}\right]}.$$

This can be set in the next form:

$$\mathbf{a}_A^{O_A}(\tau_a;\tau_{a0}) = \sqrt{1-\frac{\nu^2}{c^2}}\frac{\left(1-\dfrac{\nu^2}{c^2}\right)\mathbf{a}_A^{O_B}(\tau_b;\tau_{b0})}{\left[1-\dfrac{\nu v_A^{O_B}(\tau_b;\tau_{b0})}{c^2}\right]^3},$$

or

$$\mathbf{a}_A^{O_A}(\tau_a;\tau_{a0}) = \frac{\left(1-\dfrac{\nu^2}{c^2}\right)^{3/2}\mathbf{a}_A^{O_B}(\tau_b;\tau_{b0})}{\left[1-\dfrac{\nu v_A^{O_B}(\tau_b;\tau_{b0})}{c^2}\right]^3},$$

which implies (22a). The equation (22b) is analogously proved by starting with the definition of $\mathbf{a}_A^{O_B}(\tau_b;\tau_{b0})$, (21), and by using (15b) and (19b).

Notice that the equations (22) cannot be derived directly from the equations (20) for $c^a = c^b = c$. The equations (22) are well known in the relativity theory. They are derived herein by starting with the time definition in Newton's sense.

6. Mass and coordinate transformation. Let the example of two particles of equal masses at the rest considered by D'Inverno [52, p.45 – 47] be reconsidered, Fig. 7. They are assumed to collide inelastically.

The reference frames $R = R^n$ and $R_A = R_a^n$: The reference frames $R = R^n$ and $R_B = R_b^n$:

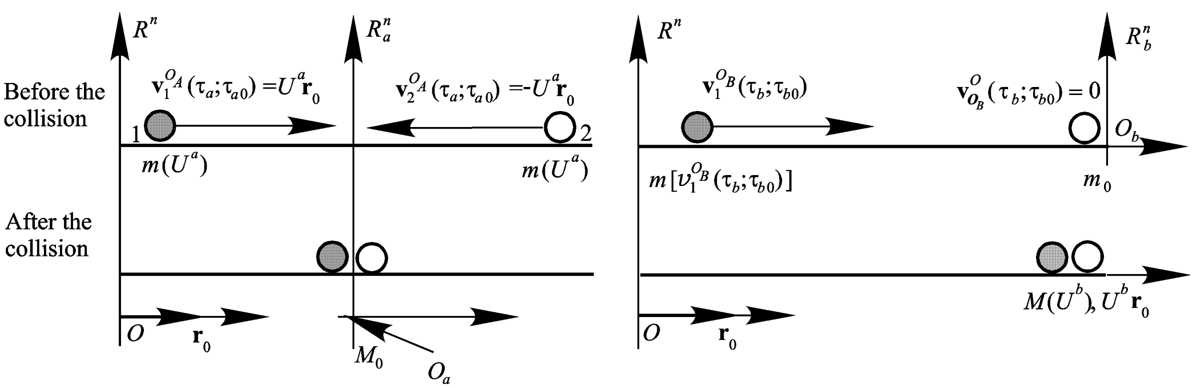

Fig. 7. Two equal mass particles before and after the collision in the reference frames R, R_A and R_B: $U^a = U(\tau_a;\tau_{a0})$, $U^b = U(\tau_b;\tau_{b0}) = \frac{c^b}{c^a}U^a$ (Corollary1), $\mathbf{v}_1^{O_B} = \mathbf{v}_1^{O_B}(\tau_b;\tau_{b0}) = v_1^{O_B}\mathbf{r}_0$.

The three inertial coordinate systems (reference frames) are accepted. The coordinate system $R = R^n$ is the reference coordinate system for other two coordinate systems $R_A = R_a^n$ and $R_B = R_b^n$, Fig. 7.

The frame R_a^n is the "centre-of-mass" frame. Both particles move relative to the frame R_a^n. Their velocities have the same absolute value U^a measured in time scale T_a and before the collision, but opposite sence. Their common speed after the collision equals zero relative to the reference frame R_a^n.

The reference frame R_b^n is tied with the particle 2. The second particle 2 is evidently at the rest also relative to R_b^n, and both are at the rest relative to R^n by accepting that the frame R_b^n does not move relative to R^n.

Both particles move evidently with the common speed U^a with respect to the frame R^n after the collision, when the speed is measured relative to T_a. Their common speed value is U^b with respect to the frame R^n after the collision if the speed is measured relative to T_b, and then Corollary 1 provides the following relationship (Fig. 7):

$$\frac{U^a}{U_b} = \frac{\mu_a}{\mu_b} = \frac{c^a}{c^b}. \tag{23}$$

The general case.

Theorem 8. *Let two identical particles 1 and 2 have the same mass m_0 at the rest relative to a reference coordinate system $T \times R^n$. Let another two coordinate systems $T_a \times R_a^n$ and $T_b \times R_b^n$ be accepted, $T_a \neq T_b$. Let the space coordinate system R_a^n be the “centre-of-mass” frame. Let the space coordinate system R_b^n be at the rest relative to R^n. Then, the mass $m(v_1^{O_B})$ of the particle 1 moving with the speed $\mathbf{v}_1^{O_B} = v_1^{O_B}\mathbf{r}_0 = v_1^{O_B}(\tau_b;\tau_{b0})\mathbf{r}_0$ is related to its mass m_0 at the rest relative to R_b^n by*

$$m(v_1^{O_B}) = \frac{m_0}{\sqrt{1 - \left(\dfrac{v_1^{O_B}}{c^b}\right)^2}}. \tag{24}$$

Proof. Let all the conditions of the theorem statement hold. Then the law of the mass conservation and the law of the linear momentum take the following forms for the two particles:

$$m(v_1^{O_B}) + m_0 = M(U^b),$$

$$m(v_1^{O_B})v_1^{O_B} + m_0 = M(U^b)U^b.$$

These equations determine $m(v_1^{O_B})$, after eliminating $M(U^b)$, as follows:

$$m(v_1^{O_B}) = \frac{U^b}{v_1^{O_B} - U^b} m_0.$$

Combining the preceding equation with (23) we can express $m(v_1^{O_B})$ in terms of U^a:

$$m(v_1^{O_B}) = \frac{\dfrac{c^b}{c^a}U^a}{v_1^{O_B} - \dfrac{c^b}{c^a}U^a} m_0, \tag{25}$$

which is suitable because we can determine U^a as follows by using (17b) in which $v_A^{O_A}(\tau_a;\tau_{a0}) = v_1^{O_A}(\tau_a;\tau_{a0}) = U^a$ and $\nu^a = U^a$:

$$v_1^{O_B} = \frac{c^b}{c^a}\frac{2U^a}{1 + \left(\dfrac{U^a}{c^a}\right)^2}.$$

This is to be solved for U^a:

$$U^a = \frac{c^a c^b}{v_1^{O_B}} \left[1 \pm \sqrt{1 - \left(\frac{v_1^{O_B}}{c^b} \right)^2} \right].$$

From the condition that $|U^a| < \infty$ should be satisfied as $v_1^{O_B} \to 0$ we conclude that the negative sigh should be retained:

$$U^a = \frac{c^a c^b}{v_1^{O_B}} \left[1 - \sqrt{1 - \left(\frac{v_1^{O_B}}{c^b} \right)^2} \right].$$

Replacing U^a by the right-hand side of this equation in the equation (25), and after simple algebraic manipulations we derive:

$$m(v_1^{O_B}) = \frac{m_0}{\sqrt{1 - \left(\frac{v_1^{O_B}}{c^b} \right)^2}},$$

which proves (24).

This generalizes the expression for the relativistic mass that exists in the relativity theory and which can be determined now easily for the special case.

The special case.

Theorem 9. *Let two identical particles 1 and 2 have the same mass m_0 at the rest relative to a reference coordinate system $T \times R^n$. Let another two coordinate systems $T_a \times R_a^n$ and $T_b \times R_b^n$ be accepted so that $T_a = T_b$. Let the space coordinate system R_a^n be the "centre-of-mass" frame. Let the space coordinate system R_b^n be at the rest relative to R^n. Then, the mass $m(v_1^{O_B})$ of the particle 1 moving with the speed $\mathbf{v}_1^{O_B} = v_1^{O_B}\mathbf{r}_0 = v_1^{O_B}(\tau_b; \tau_{b0})\mathbf{r}_0$ is related to its mass m_0 at the rest relative to R_b^n by*

$$m(v_1^{O_B}) = \frac{m_0}{\sqrt{1 - \left(\frac{v_1^{O_B}}{c} \right)^2}}. \tag{26}$$

Proof. Let all the conditions of the theorem statement hold. Since $T^a = T^b$ then by the definition $\mu_a = \mu_b$ (the equations (3)) which together with (23) implies $c^a = c^b = c$. By replacing now c^b by c in (24) we obtain (26) that is well known expression for the relativistic mass in the relativity theory.

The equations (24) and (26) have been derived by starting with the time definition in Newton's sense.

7. Force and coordinate transformation. *The general case.*

Theorem 10. *Let m_0 be the mass of a particle 1 at the rest relative to a reference coordinate system $T \times R^n$. Let another two coordinate systems $T_a \times R_a^n$ and $T_b \times R_b^n$ be accepted, $T_a \neq T_b$. Let the space coordinate system R_a^n be the "centre-of-mass" frame for the particle 1 and a particle*

2. Let the particle 2 be at the rest relative to R_b^n and let its mass be also m_0 at the rest. Let the space coordinate system R_b^n be at the rest relative to R^n. Then, a force F acting on the particle 1, its speed $\mathbf{v}_1^{O_B} = v_1^{O_B}(\tau_b; \tau_{b0})\mathbf{r}_0$ and acceleration $\mathbf{a}_1^{O_B} = a_1^{O_B}(\tau_b; \tau_{b0})\mathbf{r}_0$ relative to R_b^n are interrelated by:

$$\mathbf{F}(\mathbf{v}_1^{O_B}) = \frac{m_0}{\left[1 - \left(\dfrac{v_1^{O_B}}{c^b}\right)^2\right]^{3/2}} \mathbf{a}_1^{O_B}. \tag{27}$$

Proof. The equation (27) follows directly from Newton's law: $\mathbf{F}(\mathbf{v}_1^{O_B}) = \dfrac{d}{d\tau_b}\left[m(\mathbf{v}_1^{O_B})\,\mathbf{v}_1^{O_B}\right]$ and (24).

The special case.

Theorem 11. *Let m_0 be the mass of a particle 1 at the rest relative to a reference coordinate system $T \times R^n$. Let another two coordinate systems $T_a \times R_a^n$ and $T_b \times R_b^n$ be accepted so that $T_a = T_b$. Let the space coordinate system R_a^n be the "centre-of-mass" frame for the particle 1 and a particle 2. Let the particle 2 be at the rest relative to R_b^n ant its mass be also m_0 at the rest. Let the space coordinate system R_b^n be at the rest relative to R^n. Then, a force F acting on the particle 1, its speed $\mathbf{v}_1^{O_B} = v_1^{O_B}(\tau_b; \tau_{b0})\mathbf{r}_0$ and acceleration $\mathbf{a}_1^{O_B} = a_1^{O_B}(\tau_b; \tau_{b0})\mathbf{r}_0$ relative to R_b^n are interrelated by:*

$$\mathbf{F}(\mathbf{v}_1^{O_B}) = \frac{m_0}{\left[1 - \left(\dfrac{v_1^{O_B}}{c}\right)^2\right]^{3/2}} \mathbf{a}_1^{O_B}. \tag{28}$$

Proof. The equation (28) can be deduced directly from Newton's law: $\mathbf{F}(\mathbf{v}_1^{O_B}) = \dfrac{d}{d\tau_b}\left[m(\mathbf{v}_1^{O_B})\,\mathbf{v}_1^{O_B}\right]$ and (26), or from (27) for $c^b = c$ that holds due to $T_a = T_b$ implying $\mu_a = \mu_b$ and therefore $c^b = c$, (23).

The equations (27) and (28) have been derived by starting with the time definition in Newton's sense.

8. Energy and coordinate transformation. The expressions for the energy E of a body now result from its general expression $E = mc^2$ and the expression for the corresponding mass depending on the body velocity v^{O_B} relative to the reference frame $T_b \times R_b^n$ with respect to which the body mass at the rest is m_0.

The general case.

Theorem 12. *Let two identical particles 1 and 2 have the same mass m_0 at the rest relative to a reference coordinate system $T \times R^n$. Let another two coordinate systems $T_a \times R_a^n$ and $T_b \times R_b^n$ be accepted, $T_a \neq T_b$. Let the space coordinate system R_a^n be the "centre-of-mass" frame for the particle 1 and a particle 2. Let the particle 2 be at the rest relative to R_b^n. Let the space coordinate system R_b^n be at the rest relative to R^n. Then, the energy $E(v_1^{O_B})$ of the particle 1 moving with the speed $\mathbf{v}_1^{O_B} = v_1^{O_B}\mathbf{r}_0 = v_1^{O_B}(\tau_b; \tau_{b0})\mathbf{r}_0$ is related to its mass m_0 and to its energy $E_0 = m_0c^2$ when it is at the rest relative to R_b^n by*

$$E(v_1^{O_B}) = \frac{m_0(c^b)^2}{\sqrt{1-\left(\frac{v_1^{O_B}}{c^b}\right)^2}} = \left(\frac{c^b}{c}\right)^2 \frac{E_0}{\sqrt{1-\left(\frac{v_1^{O_B}}{c^b}\right)^2}}. \tag{29}$$

Proof. The equation (29) follows directly from $E(v_1^{O_B}) = m(v_1^{O_B})(c^b)^2$ and (24).

The special case.

Theorem 13. *Let two identical particles 1 and 2 have the same mass m_0 at the rest relative to a reference coordinate system $T \times R^n$. Let another two coordinate systems $T_a \times R_a^n$ and $T_b \times R_b^n$ be accepted so that $T_a = T_b$. Let the space coordinate system R_a^n be the "centre-of-mass" frame. Let the space coordinate system R_b^n be at the rest relative to R^n. Then, the energy $E(v_1^{O_B})$ of the particle 1 moving with the speed $\mathbf{v}_1^{O_B} = v_1^{O_B}\mathbf{r}_0 = v_1^{O_B}(\tau_b; \tau_{b0})\mathbf{r}_0$ is related to its mass m_0 and to is energy E_0 when it is at the rest relative to R_b^n by:*

$$E(v_1^{O_B}) = \frac{m_0 c^2}{\sqrt{1-\left(\frac{v_1^{O_B}}{c}\right)^2}} = \frac{E_0}{\sqrt{1-\left(\frac{v_1^{O_B}}{c}\right)^2}}. \tag{30}$$

Proof. The equation (30) can be deduced directly from $E(v_1^{O_B}) = m(v_1^{O_B})(c^b)^2$, (26) and $c^b = c$, or from (29) for $c^b = c$.

The equations (29) and (30) have been derived by starting with the time definition in Newton's sense.

9. Time, physical continuity and uniqueness principle, and control. In order for a mathematical description of a physical system to be adequate it is necessary (but not sufficient) that all the system variables obey the Physical Continuity and Uniqueness Principle, which is accepted to hold for the following mathematical model:

$$\frac{dx(t)}{dt} = f[x(t), d(t)] + B[x(t)]u(t;\cdot), \ \operatorname{rank} B(x) = n \quad \text{for all } x \in R^n, \tag{31}$$

where

$$x \in R^n, f(\cdot): R^n \times R^d \to R^n, \ d(\cdot): R \to R^d, \ B(\cdot): R^n \to R^{n\times r}, \ u(\cdot): R \times \cdots \to R^r.$$

It is demanded that every system motion $x(.; x_0; d; u)$, $x(t; x_0; d; u) \equiv x(t)$, tracks elementwise with a requested tracking quality any system desired motion $x_d(\cdot) \in S_x$ for arbitrary perturbation vector function $d(\cdot) \in S_d$. The requested tracking quality can be defined by properties of solutions to the following vector equation:

$$h\left[e^{(i)}(t), \ldots, e(t), \int_0^t e(t)dt\right] = 0$$

$$\text{for all } \ t \geq 0, \ h(\cdot): R^n \times \cdots \times R^n \to R^n, \ i \in \{1, 2, \ldots\}, \tag{32}$$

where $e(\cdot)$ is the error vector function, $e(t) = x_d(t) - x(t)$.

Real forms of the vector functions $f(\cdot)$ and $d(\cdot)$ are unknown as well as their real instantaneous vector values. However the matrix function $B(\cdot)$ ant the vector function $h(\cdot)$ are completely known. The state vector $x(t)$ is elementwise measurable.

A control is self-adaptive tracking control for system (31) if and only if it guarantees that the system exhibits with the requested quality elementwise tracking of any its desired motion $x_d(\cdot) \in S_x$ for arbitrary perturbation vector function $d(\cdot) \in S_d$, and that it can be synthesised and implemented without using any information about real forms and vector values of $f(\cdot)$ and $d(\cdot)$, but by using information about $h(\cdot)$, $e(\cdot)$, $x(\cdot)$ and possibly about a subsidiary matrix function $C(\cdot) : R^n \to R^{r\times n}$ in order to ensure an appropriate error vector to control vector vector channelling. The self-adaptive tracking control $u(\cdot)$ is natural tracking control if and only if the subsidiary matrix function $C(\cdot)$ is not used for its synthesis and implementation.
Let $u(t^-) \equiv u(t-\varepsilon)$ for ε sufficiently small : $0 < \varepsilon < 1$ or for $\varepsilon \to 0^+$.In the ideal case $\varepsilon = 0^+$.

Theorem 14. *(a) Let all variables of system (31) obey Physical Continuity and Uniqueness Principle. In order for a control $u(\cdot)$ to be in the ideal case or in the case ε is sufficiently small a self-adaptive tracking control for the system on $S_d \times S_x$ ensuring a requested tracking quality defined by (32) it is both necessary and sufficient that:*

$$u[t, e(t), x(t)] = u[t^-, e(t^-), x(t^-)]+$$

$$+C[x(t)]\ \{B[x(t)]C[x(t)]\}^{-1}\, h\left[e^{(i)}(t), \ldots, e(t), \int_0^t e(t)\, dt\right], \qquad (33)$$

where $C(\cdot) : R^n \to R^{r\times n}$ is any matrix obeying elementwise the Physical Continuity and Uniqueness Principle and $\det B(x)C(x) \neq 0$ for all $x \in R^n$. $C(x) \equiv B^T(x)$ is the additional necessary and sufficient condition for the control $u(\cdot)$ (33) to be the natural tracking control.

Proof. *Necessity*. Let all variables of system (31) obey the Physical Continuity and Uniqueness Principle and let a control $u(\cdot)$ be (in the ideal case or in the case ε is sufficiently small) a self-adaptive tracking control for system (31) on $S_d \times S_x$ ensuring a requested tracking quality defined by (32). The equation (31) can be written also as:

$$\frac{dx(t^-)}{dt} = f[x(t^-), d(t^-)] + B[x(t^-)]\, u(t^-; \cdot),$$

or together with (32),

$$\frac{dx(t^-)}{dt} = f[x(t^-), d(t^-)] + B[x(t^-)]\, u(t^-; \cdot) + h\left[e^{(i)}(t), \ldots, e(t), \int_0^t e(t)\, dt\right].$$

The equation (31) can be subtracted from this equation:

$$\frac{dx(t^-)}{dt} - \frac{dx(t)}{dt} = f[x(t^-), d(t^-)] - f[x(t), d(t)] + B[x(t^-)]\, u(t^-; \cdot)-$$

$$- B[x(t)]\, u(t; \cdot) + h\left[e^{(i)}(t), \ldots, e(t), \int_0^t e(t)\, dt\right]. \qquad (34)$$

Since all the variables obey the Physical Continuity and Uniqueness Principle then, in the ideal case or in the case ε is sufficiently small, due to time continuity:

$$\frac{dx(t^-)}{dt} \equiv \frac{dx(t)}{dt} \text{ and } f[x(t^-), d(t^-)] \equiv f[x(t), d(t)]$$

so that equation (34) reduces to:

$$B[x(t)]\, u(t;\cdot) = B[x(t^-)]\, u(t^-;\cdot) + h\left[e^{(i)}(t), \ldots, e(t), \int\limits_0^t e(t)\, dt\right]. \tag{35}$$

Let $C(\cdot) : R^n \to R^{r\times n}$ be such that $\det B(x)C(x) \neq 0$ for all $x \in R^n$ that is possible due to $\det B(x) \neq 0$ for all $x \in R^n$, and let $u(t;\cdot) = C[x(t)]w(t;\cdot)$ so that $w(;\cdot)$ well defines $u(t;\cdot)$. Such a determination of $u(t;\cdot)$ and the definition of $C(\cdot)$ transform (35) into

$$w[t, e(t), x(t)] = \{B[x(t)]C[x(t)]\}^{-1} \left\{B[x(t^-)]C[x(t^-)]\right\} w[t^-, e(t^-), x(t^-)] +$$
$$+ \{B[x(t)]C[x(t)]\}^{-1}\, h\left[e^{(i)}(t), \ldots, e(t), \int\limits_0^t e(t)\, dt\right],$$

which can be set in the next form implied by $B[x(t)]C[x(t)] \equiv B[x(t^-)]C[x(t^-)]$ ensured by the Physical Continuity and Uniqueness Principle, by time continuity and by non-singularity of $B(\cdot)C(\cdot)$:

$$w[t, e(t), x(t)] = w[t^-, e(t^-), x(t^-)] +$$
$$+ \{B[x(t)]C[x(t)]\}^{-1}\, h\left[e^{(i)}(t), \ldots, e(t), \int\limits_0^t e(t)\, dt\right].$$

After multiplying this equation by $C[x(t)]$ on the left and by using $u(t;\cdot) = C[x(t)]w(t;\cdot)$ we prove (33). Let now the control $u(\cdot)$ (33) be natural tracking control. Hence, an additional matrix $C(x)$ is not used, which implies $C(x) \equiv B^T(x)$.

Sufficiency. Let all conditions ant the statement of the theorem to be proved hold. Then (33) and (31) result in:

$$\frac{dx(t)}{dt} = f[x(t), d(t)] + B[x(t^-)]\, u(t^-;\cdot) + h\left[e^{(i)}(t), \ldots, e(t), \int\limits_0^t e(t)\, dt\right]$$

or, by using (31) for t^- in order to replace $B[x(t^-)]u(t^-)$ by $\dfrac{dx(t^-)}{dt} - f[x(t^-), d(t^-)]$ in the preceding equation, we find:

$$\frac{dx(t)}{dt} = f[x(t), d(t)] + \frac{dx(t^-)}{dt} - f[x(t^-), d(t^-)] + h\left[e^{(i)}(t), \ldots, e(t), \int\limits_0^t e(t)\, dt\right],$$

which results in:

$$0 = h\left[e^{(i)}(t), \ldots, e(t), \int\limits_0^t e(t)\,dt\right], \tag{36}$$

due to

$$\frac{dx(t^-)}{dt} \equiv \frac{dx(t^-)}{dt} \text{ and } f[x(t^-, d(t^-)] \equiv f[x(t), d(t)]$$

guaranteed by the Physical Continuity and Uniqueness Principle, by continuity of time and by sufficiently small ε. The equation (36) proves that the system exhibits the requested tracking property. Evidently, $C(x) \equiv B^T(x)$ can be accepted due to rank $B(x) \equiv n$. In such a case an additional matrix $C(x)$ is not used to synthesise the control $u(\cdot)$ (33), which is therefore the natural tracking control for the system.

10. Conclusion. The essential features of time are explained in Newton's sence: time is a physical variable the value of which (called: moment or instant) has been strictly monotonously increasing independently of all other variables, processes, movements and events. For this reason Newton explained the absolute sense and meaning of time. However, *Newton himself* explained also a relative sense, meaning and use of time by introducing different time units and scales in order to measure the time values and their difference (time intervals). The time values are used to determine the order of happenings of events, processes and/or movements. A time interval is used to measure a duration of a process, of a movement or of a rest. Time evolution expresses the ageing process. Ageing speeds can be different not only for different processes, objects, systems, but also for different parts of the same process, object, system. Different ageing speeds can imply different time scales and units. This can create an incorrect impression, feelingt and/or interpretation of "different times" instead of the correct conclusion: "different time units and/or different time scales".

Such Newton's understanding of time was used at first to generalize the Lorentz transformation [65, 66], which has bee exploited herein to prove new results on velocity, acceleration, mass and energy of a moving body, and on a force causing its movement. The new relationships are more general than those existing in the relativity theory. The former incorporates the latter. It has been also shown that by using the real features of time as explained essentially by Newton and in more details herein, we are able to deduce all the basic relationships of the relativity theory and to show that they correspond to a special, singular case.

Moreover, such Newton's understanding of time gives a fundamental importance to a new physical principle: Physical Continuity and Uniqueness Principle discovered recently [67 – 70] and explained in the paper. Their combination is the basis of a new concept of control of technical objects, which just reflects how the Nature creates a control. The essence of such a control called natural (tracking) control, or self-adaptive control, is explained in the paper by proving the corresponding control algorithm. One of its characteristics is the use of information about the error by closing a global negative feedback and about the just realised control by closing a local positive unit feedback. The former ensures a requested tracking property to the controlled system, i.e. it guarantees the requested tracking quality. The latter compensates unknown, unpredictable variations of the object internal dynamics and of actions of also unknown, unpredictable external disturbances. Hence, the natural (tracking) controller does not use any information about the mathematical model of the object internal dynamics and about external disturbances in order to generate the control. As the Nature does.

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Received 13.04.99