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EXISTENCE OF A CONTINUOUSLY DIFFERENTIABLE SOLUTION OF A CAUCHY PROBLEM FOR A SYSTEM OF INTEGRO-FUNCTIONAL EQUATIONS WITH PARTIAL DERIVATIVES AND LINEARLY TRANSFORMED ARGUMENTS

M. I. Gromyak

Ternopil' State Pedagogical University, Ukraine

A theorem of existence of continuously differentiable solution of a system of integro-functional equations with partial derivatives and linearly transformed arguments is proved.

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We consider a system of nonlinear differential equations

$$u_t - \Lambda u_x = f(t, x, u(x,t), u(\lambda t, x), u(\lambda t, \mu x),$$

$$\int_0^{h(t,x)} \psi(t, x, s, u(s,x), u(\lambda s, x), u(\lambda s, \mu x))ds), \tag{1}$$

where $\Lambda = \mathrm{diag}(\lambda_1, \ldots, \lambda_n)$, λ_i, $i = \overline{1,n}$, are real numbers, $\lambda, \mu \in \mathbb{R}$ $(\lambda\mu \neq 0)$, $t,\ x$ belong to some closed domain $\overline{D}$, $f(t, x, v_1, v_2, v_3, v_4) : \overline{D} \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^1 \to \mathbb{R}^n$, where

$$v_4 = \int_0^{h(t,x)} \psi(t, x, s, u(s,x), u(\lambda s, x), u(\lambda s, \mu x))ds,$$

$u(t,x)$ is an unknown vector-valued function.

Denote by $\lambda_* = \min\limits_{1\le i\le n} \lambda_i$, $\lambda^* = \max\limits_{1\le i\le n} \lambda_i$, and assume that the domain $\overline{D}$ is bounded by a segment $[0,a]$ on the axis OX $(a = \text{const} > 0)$ and characteristics l_1 and l_2, with angular coefficients $1/\lambda^*$ and $1/\lambda_*$, starting in the points 0 and a, respectively.

The aim of this article is to study the problem of existence of a solution of the system such that the solution has a continuous derivative of the first order in $\overline{D}$. Also, this solution should satisfy the initial condition

$$u_i(0,x) = \varphi_i(x), \tag{2}$$

where $\varphi(x)$ is a vector-valued function continuously differentiable in the segment $[0,a]$.

Theorem 1. *Let the following conditions hold:*
1) $\lambda^* \cdot \lambda_* < 0$ *and* $0 < \lambda \le \mu < 1$*;*

2) $$\sup_{(t,x)\in\overline{D}} \left| f(t,x,0,0,0,\int_0^{h(t,x)} \psi(t,x,s,0,0,0)ds) \right|$$

$$= \sup_{(t,x)\in\overline{D}} \max_{1\le i\le n} \left| f_i(t,x,0,0,0,\int_0^{h(t,x)} \psi(t,x,s,0,0,0)ds) \right| = M_2 < \infty,$$

$$\sup_{(t,x)\in\overline{D}} |\psi(t,x,s,0,0,0)| = \sup_{(t,x)\in\overline{D}} \max_{1\le i\le n} |\psi_i(t,x,s,0,0,0)| = M_3 < \infty,$$

$$|h(t,x)| \le N, \qquad |\varphi(x)| \le M_1;$$

3) *the vector-valued functions* $f(t,x,v_1,v_2,v_3,v_4,$ $\psi(t,x,s,v_1,v_2,v_3)$ *and their partial derivatives,*

$$\frac{\partial^{i_1+i_2} f(t,x,v_1,v_2,v_3,v_4)}{\partial x^{i_1}\partial v_j^{i_2}}, \qquad \frac{\partial^{i_1+i_2} \psi(t,x,s,v_1,v_2,v_3)}{\partial x^{i_1}\partial v_k^{i_2}},$$

$$i_1 + i_2 = 1, \quad j = \overline{1,4}, \quad k = \overline{1,3},$$

are continuous with respect to all variables for every $(t,x) \in \overline{D}$*,* $v_1 \in \mathbb{R}^n$*,* $v_2 \in \mathbb{R}^n$*,* $v_3 \in \mathbb{R}^n$*,* $v_4 \in \mathbb{R}^1$ *and satisfy the Lipschitz condition*

$$|f(t,x,v_1',v_2',v_3',v_4') - f(t,x,v_1'',v_2'',v_3'',v_4'')|$$

$$\le l(|v_1'-v_1''| + |v_2'-v_2''| + |v_3'-v_3''| + |v_4'-v_4''|),$$

$$\left| \frac{\partial^{i_1+i_2} f(t,x,v_1',v_2',v_3',v_4')}{\partial x^{i_1}\partial v_j^{i_2}} - \frac{\partial^{i_1+i_2} f(t,x,v_1'',v_2'',v_3'',v_4'')}{\partial x^{i_1}\partial v_j^{i_2}} \right|$$

$$\le l(|v_1'-v_1''| + |v_2'-v_2''| + |v_3'-v_3''| + |v_4'-v_4''|),$$

$$|\psi(t,x,s,v_1',v_2',v_3' - \psi(t,x,s,v_1'',v_2'',v_3''|$$

$$\le l^*(|v_1'-v_1''| + |v_2'-v_2''| + |v_3'-v_3''|,$$

$$\left|\frac{\partial^{i_1+i_2}\psi(t,x,s,v_1',v_2',v_3')}{\partial x^{i_1}\partial v_j^{i_2}} - \frac{\partial^{i_1+i_2}\psi(t,x,s,v_1'',v_2'',v_3'')}{\partial x^{i_1}\partial v_j^{i_2}}\right| \leq$$

$$\leq l^*(|v_1'-v_1''|+|v_2'-v_2''|+|v_3'-v_3''|),$$

where $l, l^* = \text{const} > 0$,

$$(t,x,v_1',v_2',v_3',v_4'),(t,x,v_1'',v_2'',v_3'',v_4'') \in \overline{D}\times\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^1 \to \mathbb{R}^n,$$

$$(t,x,s,v_1',v_2',v_3'),(t,x,s,v_1'',v_2'',v_3'') \in \overline{D}\times\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^1 \to \mathbb{R}^n.$$

Then there exists a continuously differentiable solution of the problem (1), (2) in the domain $\overline{D}$.

Proof. First of all it is sufficient to show that there exists a solution of the system of integral equations of the form

$$u_i(t,x) = \varphi_i(x-\lambda_i t) + \int_0^t f_i(\tau,\lambda_i(\tau-t)+x, u(\tau,\lambda_i(\tau-t)+x),$$

$$u(\lambda\tau,\lambda_i(\tau-t)+x, u(\lambda\tau,\mu(\lambda_i(\tau-t)+x)),$$

$$\int_0^{h(t,x)} \psi(\tau,\lambda_i(\tau-t)+x),s,u(s,\lambda_i(\tau-t)+x),u(\lambda s,\lambda_i(\tau-t)+x),$$

$$u(\lambda s,\mu(\lambda_i(\tau-t)+x))ds))d\tau, \tag{3}$$

which is continuously differentiable with respect to t and x in $\overline{D}$.

In order to construct a solution of the system (3), we can use the method of successive approximations. The successive approximations $u_i^m, i = \overline{1,n}, m = 0,1,\ldots$, are defined the following relations:

$$u_i^0(t,x) = 0,$$

$$u_i^m(t,x) = \varphi_i(x-\lambda_i t) + \int_0^t f_i(\tau,\lambda_i(\tau-t)+x, u^{m-1}(\tau,\lambda_i(\tau-t)+x),$$

$$u^{m-1}(\lambda\tau,\lambda_i(\tau-t)+x, u^{m-1}(\lambda\tau,\mu(\lambda_i(\tau-t)+x)),$$

$$\int_0^{h(\tau,x-\lambda_i t)} \psi(\tau, \lambda_i(\tau - t) + x), s, u^{m-1}(s, \lambda_i(\tau - t) + x), u^{m-1}(\lambda s, \lambda_i(\tau - t) + x),$$

$$u^{m-1}(\lambda s, \mu(\lambda_i(\tau - t) + x))ds))d\tau, \qquad i = \overline{1, n}, \quad m = 1, 2, \ldots. \tag{4}$$

We shall show that the sequences of the continuous functions $u_i^m(t, x)$, $i = \overline{1, n}, m = 0, 1, \ldots$, uniformly converge to some continuous functions $u_i(t, x)$, $i = \overline{1, n}$, for every $(t, x) \in \overline{D}$.

It is sufficient to show that the following estimates hold:

$$|u_i^m(t, x) - u_i^{m-1}(t, x)| \le M \frac{(3lQt)^{m-1}}{(m-1)!}, \ i = \overline{1, n}, \ \text{where } M, Q = \text{const} > 0, \tag{5}$$

for every $(t, x) \in \overline{D}$ and $m \ge 1$.

Condition 2), for $m = 1$ and (3), implies that

$$|u_i^1(t, x) - u_i^0(t, x)| \le |\varphi_i(x - \lambda_i t)| + \int_0^t \Bigg| f_i(\tau, \lambda_i(\tau - t) + x, 0, 0, 0,$$

$$\int_0^{h(\tau,x-\lambda_i t)} \psi(\tau, \lambda_i(\tau - t) + x), s, 0, 0, 0)ds \Bigg| d\tau \le M_1 + M_2 t = M.$$

In this case the condition (5) is satisfied.

Suppose that the condition (5) holds for some $m \ge 1$ and show that it will be the same if we pass from m to $m + 1$. Indeed, in view of (4), 2), 3), and (5), we have

$$|u_i^m(t, x) - u_i^{m-1}(t, x)| \le \int_0^t \Big| f_i(\tau, \lambda_i(\tau - t) + x, u^m(\tau, \lambda_i(\tau - t) + x),$$

$$u^m(\lambda\tau, \lambda_i(\tau - t) + x), u^m(\lambda\tau, \mu(\lambda_i(\tau - t) + x)),$$

$$\int_0^{h(\tau,x-\lambda_i t)} \psi(\tau, \lambda_i(\tau - t) + x), s, u^m(s, \lambda_i(\tau - t) + x), u^m(\lambda s, \lambda_i(\tau - t) + x),$$

$$u^m(\lambda s, \mu(\lambda_i(\tau - t) + x))ds) - f_i(\tau, \lambda_i(\tau - t) + x, u^{m-1}(\tau, \lambda_i(\tau - t) + x),$$

$$u^{m-1}(\lambda\tau,\lambda_i(\tau-t)+x),u^{m-1}(\lambda\tau,\mu(\lambda_i(\tau-t)+x)),$$

$$\int\limits_0^{h(t,x)}\psi(\tau,\lambda_i(\tau-t)+x),s,u^{m-1}(s,\lambda_i(\tau-t)+x),u^{m-1}(\lambda s,\lambda_i(\tau-t)+x),$$

$$u^{m-1}(\lambda s,\mu(\lambda_i(\tau-t)+x))ds)\Big|d\tau$$

$$\leq\int\limits_0^t l(|u^m(\tau,\lambda_i(\tau-t)+x)-u^{m-1}(\tau,\lambda_i(\tau-t)+x)|$$

$$+\ |u^m(\lambda\tau,\lambda_i(\tau-t)+x)-u^{m-1}(\lambda\tau,\lambda_i(\tau-t)+x)|$$

$$+\ |u^m(\lambda\tau,\mu(\lambda_i(\tau-t)+x))-u^{m-1}(\lambda\tau,\mu(\lambda_i(\tau-t)+x))|$$

$$+\int\limits_0^{h(\tau,x-\lambda_i t)} l^*(|u^m(s,\lambda_i(\tau-t)+x)-u^{m-1}(s,\lambda_i(\tau-t)+x)|$$

$$+\ |u^m(\lambda s,\lambda_i(\tau-t)+x)-u^{m-1}(\lambda s,\lambda_i(\tau-t)+x)|$$

$$+\ |u^m(\lambda s,\mu(\lambda_i(\tau-t)+x))-u^{m-1}(\lambda s,\mu(\lambda_i(\tau-t)+x))|)ds)d\tau\leq M\frac{(3lQt)^m}{m!},$$

where $Q=1+l^*N$. Thus, condition (5) is satisfied for every $(t,x)\in\overline{D}$ and $m\geq 1$. From this it follows that the series

$$\sum_{m=1}^{\infty}(u_i^m(t,x)-u_i^{m-1}(t,x)),\qquad i=\overline{1,n},$$

and, the sequences $u_i^m(t,x),i=\overline{1,n}$, are uniformly convergent to some continuous functions $u_i(t,x),i=\overline{1,n}$, with respect to t,x for every $(t,x)\in\overline{D}$.

Passing to the limit as $m\to\infty$ in (4), we can verify that the functions $u_i(t,x),i=\overline{1,n}$, are solution of the system (3).

Let us prove that the obtained solution is continuously differentiable with respect to t,x for every $(t,x)\in\overline{D}$. It is sufficient to show that the sequences of functions $\partial u_i^m(t,x)/\partial x,\partial u_i^m(t,x)/\partial t,i=\overline{1,n},m=0,1,\ldots$, are uniformly convergent to some continuous functions $u_i^m(t,x)$ with respect to t,x for every $(t,x)\in\overline{D}$.

In view of (4), we have

$$\frac{\partial u_i^0(t,x)}{\partial x} = 0,$$

$$\frac{\partial u_i^m(t,x)}{\partial x} = \varphi'(x-\lambda_i t) + \int_0^t \Bigg(\frac{\partial f_i(m-1)}{\partial x} + \frac{\partial f_i(m-1)}{\partial v_1} \frac{\partial u^{m-1}(\tau, \lambda_i(\tau - t) + x)}{\partial x}$$

$$+ \frac{\partial f_i(m-1)}{\partial v_2} \frac{\partial u^{m-1}(\lambda\tau, \lambda_i(\tau - t) + x)}{\partial x}$$

$$+ \mu \frac{\partial f_i(m-1)}{\partial v_3} \frac{\partial u^{m-1}(\lambda\tau, \mu(\lambda_i(\tau - t) + x))}{\partial x}$$

$$+ \frac{\partial f_i(m-1)}{\partial v_4} \Bigg(\int_0^{h(\tau, x-\lambda_i - t)} \Bigg(\frac{\partial \psi_i(m-1)}{\partial x} + \frac{\partial \psi_i(m-1)}{\partial v_1} \frac{\partial u^{m-1}(s, \lambda_i(\tau - t) + x)}{\partial x}$$

$$+ \frac{\partial f_i(m-1)}{\partial v_2} \frac{\partial u^{m-1}(\lambda s, \lambda_i(\tau - t) + x)}{\partial x}$$

$$+ \mu \frac{\partial \psi_i(m-1)}{\partial v_3} \frac{\partial u^{m-1}(\lambda s, \mu(\lambda_i(\tau - t) + x))}{\partial x} \Bigg) ds \Bigg) \Bigg) d\tau,$$

$$\frac{\partial u_i^0(t,x)}{\partial t} = 0,$$

$$\frac{\partial u_i^m(t,x)}{\partial t} = -\lambda_i \varphi'(x - \lambda_i t) + f_i(\tau, \lambda_i(\tau - t) + x, u^{m-1}(\tau, \lambda_i(\tau - t) + x),$$

$$u^{m-1}(\lambda\tau, \lambda_i(\tau - t) + x), u^{m-1}(\lambda\tau, \mu(\lambda_i(\tau - t) + x)),$$

$$\int_0^{h(\tau, x-\lambda_i t)} \psi_i(\tau, \lambda_i(\tau - t) + x, s, u^{m-1}(s, \lambda_i(\tau - t) + x),$$

$$u^{m-1}(\lambda s, \lambda_i(\tau - t) + x), u^{m-1}(\lambda s, \mu(\lambda_i(\tau - t) + x)))ds)$$

$$- \lambda_i \int_0^t \Bigg(\frac{\partial f_i(m-1)}{\partial x} + \frac{\partial f_i(m-1)}{\partial v_1} \frac{\partial u^{m-1}(\tau, \lambda_i(\tau - t) + x)}{\partial x}$$

$$+ \frac{\partial f_i(m-1)}{\partial v_2} \frac{\partial u^{m-1}(\lambda\tau, \lambda_i(\tau - t) + x)}{\partial x}$$

$$+ \mu \frac{\partial f_i(m-1)}{\partial v_3} \frac{\partial u^{m-1}(\lambda\tau, \mu(\lambda_i(\tau - t) + x))}{\partial x}$$

$$+ \frac{\partial f_i(m-1)}{\partial v_4} \Bigg(\psi_i(\tau, \lambda_i(\tau - t) + x, s, u^{m-1}(s, \lambda_i(\tau - t) + x),$$

$$u^{m-1}(\lambda s, \lambda_i(\tau - t) + x), u^{m-1}(\lambda s, \mu(\lambda_i(\tau - t) + x)))$$

$$+ \int_0^{h(\tau, x - \lambda_i t)} \Bigg(\frac{\partial \psi_i(m-1)}{\partial x} + \frac{\partial \psi_i(m-1)}{\partial v_1} \frac{\partial u^{m-1}(s, \lambda_i(\tau - t))}{\partial x}$$

$$+ \frac{\partial \psi_i(m-1)}{\partial v_2} \frac{\partial u^{m-1}(\lambda s, \lambda_i(\tau - t))}{\partial x}$$

$$+ \mu \frac{\partial \psi_i(m-1)}{\partial v_3} \frac{\partial u^{m-1}(\lambda s, \mu(\lambda_i(\tau - t)))}{\partial x} \Bigg) ds \Bigg) \Bigg) d\tau,$$

where

$$f_i(m-1) = \psi_i(\tau, \lambda_i(\tau - t) + x, u^{m-1}(\tau, \lambda_i(\tau - t) + x), u^{m-1}(\lambda\tau, \lambda_i(\tau - t) + x),$$

$$u^{m-1}(\lambda\tau, \mu(\lambda_i(\tau - t) + x)), \int_0^{h(\tau, x - \lambda_i t)} \psi(\tau, \lambda_i(\tau - t) + x, s, u^{m-1}(\tau, \lambda_i(\tau - t) + x),$$

$$u^{m-1}(\lambda\tau, \mu(\lambda_i(\tau - t) + x)))ds).$$

Now, it is sufficient to prove that only sequences

$$\frac{\partial u_i^m(t, x)}{\partial x}, \qquad i = \overline{1, n}, \ m = 0, 1, \dots ,$$

are uniformly convergent for every $(t,x) \in \overline{D}$.

Let us prove that the following relations hold:

$$\left|\frac{\partial u_i^m(t,x)}{\partial x} - \frac{\partial u_i^{m-1}(t,x)}{\partial x}\right| \leq Z\frac{(KtQ)^{m-1}}{(m-1)!}, \tag{6}$$

for every $(t,x) \in \overline{D}$ and $m \geq 1$, where Q, K, Z are some positive numbers.

As a consequence of (5), we have

$$|u_i^m(t,x)| \leq |u_i^0(t,x)| + |u_i^1(t,x) - u_i^0(t,x)|$$

$$+ \ldots + |u_i^m(t,x)| - |u_i^{m-1}(t,x)| \leq \sum_{j=1}^{\infty} |u_i^j(t,x)| - |u_i^{j-1}(t,x)|$$

$$\leq M \sum_{j=1}^{\infty} \frac{(3lTQ)^{j-1}}{(j-1)!}, \qquad i = \overline{1,n},$$

for every $m \geq 1$ and $(t,x) \in \overline{D}$, where T is some positive number larger than an arbitrary value of t from the bounded domain $\overline{D}$ and such that the number series $\sum_{j=1}^{\infty} \frac{(3lTQ)^{j-1}}{(j-1)!}$ is convergent. Then we have

$$|u_i^m(t,x)| \leq \widetilde{M}, i = \overline{1,n}, \widetilde{M} = \text{const} > 0, \tag{7}$$

for every $(t,x) \in \overline{D}$ and $m \geq 1$.

Analogously, if relations (6) hold, then we have the following inequalities for every $(t,x) \in \overline{D}$:

$$\left|\frac{\partial u_i^m(t,x)}{\partial x}\right| \leq \widetilde{N}, i = \overline{1,n}, \widetilde{N} = \text{const} > 0. \tag{8}$$

Denote by

$$L_1 = \sup_D \max_{i,j} \left|\frac{\partial f_i(t,x,v_1,v_2,v_3,v_4)}{\partial v_j}\right|, \tag{9}$$

$$L^* = \sup_D \max_{i,j} \left|\frac{\partial \psi_i(t,x,s,v_1,v_2,v_3)}{\partial v_j}\right|, \tag{10}$$

$$\overline{N} = \sup_D \max_{i} \left|\frac{\partial u_i^m(t,x)}{\partial x}\right|, \tag{11}$$

$$L^{**} = \sup_D \max_i \left| \frac{\partial v_4}{\partial x} \right|, \tag{12}$$

where $D = \{(t, x, v_1, v_2, v_3, v_4) : (t, x) \in \overline{D}, |v_1| \leq \widetilde{M}, |v_2| \leq \widetilde{M}, |v_3| \leq \widetilde{M}, |v_3| \leq S\}$.

Taking into account (1) and (9) we have

$$\left| \frac{\partial u_i^1(t,x)}{\partial x} - \frac{\partial u_i^0(t,x)}{\partial x} \right| \leq |\psi_i'(x - \lambda_i t)|$$

$$+ \int_0^t \left| \frac{\partial f_i}{\partial x}(\tau, \lambda_i(\tau - t) + x, 0, 0, 0, \frac{\partial f_i}{\partial v_4} \int_0^{h(\tau, x - \lambda_i t)} \frac{\partial \psi_i}{\partial x}(\tau, \lambda_i(\tau - t) + x, s, 0, 0, 0) ds) \right| d\tau$$

$$\leq L_2 + L_1 t < L_2 + L_1 T = Z,$$

therefore, condition (6) holds for $m = 1$. Reasoning by induction, suppose that condition (6) is proved for some $m \geq 1$ and show that this condition is the same after passing from m to $m+1$.

Indeed, using (5), conditions 2) and 3) of the theorem and (4), (5), (9), (11) we have

$$\left| \frac{\partial u_i^m(t,x)}{\partial x} - \frac{\partial u_i^m(t,x)}{\partial x} \right|$$

$$\leq 3lM \frac{(3lQ)^{m-1}}{(m-1)!} \int_0^t \tau^{m-1} d\tau + 3ll^* MN \frac{(3lQ)^{m-1}}{(m-1)!} \int_0^t \tau^{m-1} d\tau$$

$$+ (3l\overline{N} + L^{**})(3M \frac{(3lQ)^{m-1}}{(m-1)!} \int_0^t \tau^{m-1} d\tau + 3l^* NM \frac{(3lQ)^{m-1}}{(m-1)!} \int_0^t \tau^{m-1} d\tau$$

$$+ 3L_1 Z \frac{(KQ)^{m-1}}{(m-1)!} \int_0^t \tau^{m-1} d\tau + L_1 Z \frac{(KQ)^{m-1}}{(m-1)!} \int_0^t \tau^{m-1} d\tau$$

$$= Z \frac{(QKt)}{m!} \left(\frac{M}{ZQ} \left(\frac{3l}{K} \right)^m + \frac{(NMl^*)}{ZQ} \left(\frac{3l}{K} \right)^m + \frac{3\overline{N}M}{ZQ} \left(\frac{3l}{K} \right)^m \right.$$

$$\left. + \frac{3\overline{N}MNl^*}{ZQ} \left(\frac{3l}{K} \right)^m + \frac{L^{**}M}{ZQ} \left(\frac{3l}{K} \right)^m + \frac{L^{**}l^*NM}{ZQ} \left(\frac{3l}{K} \right)^m + \frac{3L_1}{QK} + \frac{L_1}{QK} \right).$$

From this it follows that, for sufficiently large K, Q, the inequality

$$\frac{M}{ZQ}\left(\frac{3l}{K}\right)^m + \frac{NMl^*}{ZQ}\left(\frac{3l}{K}\right)^m + \frac{3\overline{N}M}{ZQ}\left(\frac{3l}{K}\right)^m + \frac{3\overline{N}MNl}{ZQ}\left(\frac{3l}{K}\right)^m$$

$$+ \frac{L^{**}M}{ZQ}\left(\frac{3l}{K}\right)^m + \frac{L^{**}l^*NM}{ZQ}\left(\frac{3l}{K}\right)^m + \frac{3L_1}{QK} + \frac{L_1}{QK} < 1$$

holds and, therefore, condition (6) is true.

From (6) it directly follows that the sequences $\partial u_i^m(t,x)/\partial x, i = \overline{1,n}, m = 0,1,\ldots,$ are uniformly convergent to continuous functions $\partial u_i(t,x)/\partial x, i = \overline{1,n}$, for every $(t,x) \in \overline{D}$. This completes the proof of theorem.

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