Construction and classification of indecomposable finite-dimensional representations of the homogeneous Galilei group *)

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We discuss finite-dimensional representations of the homogeneous Galilei group which, when restricted to its subgroup SO(3), are decomposed to spin 0, $\frac{1}{2}$ and 1 representations. In particular we explain how these representations were obtained in our paper (M. de Montigny et al.: J. Phys. A **39** (2006) 9365) via reduction of the classification problem to a matrix one admitting exact solutions, and via contraction of the corresponding representations of the Lorentz group. Finally, for discussed representations we derive all functional invariants.

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1 Introduction

The Galilei group and its representations play fundamental role in non-relativistic physics. They replace the Poincaré group and its representations of relativistic physics, whenever the involved velocities are much smaller then the velocity of light.

The relativity principle of non-relativistic physics was formulated by Galileo Galilei in 1632 almost three centuries prior to the relativity principle of Albert Einstein. However, representations of the Poincaré group (the group of motions of relativistic physics) [1] have been described earlier then those of the Galilei one (see [2] and for more physical (i.e. projective) representations [3]). A masterful overview of these representations and of their possible applications was written by Lévy-Leblond [4] (see also [5]). The pivotal role in construction of physical models satisfying the Galilei relativity principle is played by finite-dimensional representations of the homogeneous Galilei group HG(1,3). But they have not been classified till now due to the fact that the structures of the Galilei group and its representations are in many respects more complicated than in the Poincaré case.

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2 Algebra of HG(1, 3) and its representation basis

The Galilei group $G(1,3)$ consists of the following transformations of time variable t and of space variables $\mathbf{x} = (x_1, x_2, x_3)$:

$$\begin{aligned} &t \to t' = t + a\,, \\ &\mathbf{x} \to \mathbf{x}' = \mathbf{R}\mathbf{x} + \mathbf{v}t + \mathbf{b}\,, \end{aligned} \tag{1}$$

where a, $\mathbf{b}$, and $\mathbf{v}$ are real parameters of time translations, space translations, and pure Galilei transformations, respectively, and matrix $\mathbf{R}$ specifies rotations determined by three parameters, namely by θ_1, θ_2 and θ_3.

The Galilei group contains a subgroup leaving invariant the point $\mathbf{x} = (0, 0, 0)$ at time $t = 0$. It is formed by all space rotations and pure Galilei transformations, i.e. by transformations (1) with $a = \mathbf{b} \equiv 0$. This subgroup is said to be *the homogeneous Galilei group* $HG(1,3)$. It is a semi-direct product of the three-parameter commutative group of pure Galilei transformations with the rotation group. Thus this group is not compact and hence has no (non-trivial) unitary finite-dimensional representations.

The Lie algebra $hg(1,3)$ of the homogeneous Galilei group has six basis elements which satisfy the following commutation relations:

$$\begin{aligned} [S_a, S_b] &= \mathrm{i}\varepsilon_{abc} S_c\,, \\ [\eta_a, S_b] &= \mathrm{i}\varepsilon_{abc}\eta_c\,, \end{aligned} \tag{2}$$

$$[\eta_a, \eta_b] = 0\,, \tag{3}$$

where indices a, b, and c take the values 1, 2, and 3 and sumation convention is used.

Algebra (2) has a commutative ideal, whose basis elements are η_1, η_2, η_3 and therefore is not compact. However, it contains the compact subalgebra so(3) with basis spanned by elements S_1, S_2, and S_3. Hermitian, finite-dimensional representations of this subalgebra are pretty well known so that it is convenient to search for representations of $hg(1,3)$ in the so(3) basis formed by eigenvectors $|s, m, \lambda\rangle$ of the commuting operators $S^2 = S_1^2 + S_2^2 + S_3^2$ and S_3 such that

$$S^2|s, m, \lambda_s\rangle = s(s+1)|s, m, \lambda_s\rangle\,, \quad S_3|s, m, \lambda_s\rangle = m|s, m, \lambda_s\rangle\,, \tag{4}$$

where s are integers or half-integers, labeling irreducible representations of so(3), $m = -s, -s+1, \ldots, s$, and λ is an additional quantum number, labeling degenerate representations of so(3) with a fixed s.

Let $\tilde{s}$ be the highest value of s appearing in (4). We restrict ourselves to the cases when $\tilde{s} = \frac{1}{2}$ and $\tilde{s} = 1$. It happens that for other values of $\tilde{s}$ the problem of description of indecomposable representations of algebra $hg(1,3)$ includes a subproblem to classify pairs of general matrices and so it is a wild linear algebraic problem (see [6]).

3 Indecomposable spinor and vector representations

Let us start with the case $\tilde{s} = \frac{1}{2}$. The corresponding matrices S_1, S_2, and S_3 can be decomposed to a direct sum of irreducible representations $D(\frac{1}{2})$ of algebra so(3):

$$S_a = \tfrac{1}{2}\, I_{n\times n} \otimes \sigma_a\,, \tag{5}$$

where $I_{n\times n}$ is the $n \times n$ unit matrix with a finite n and σ_a are the usual Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

From (5) and (2), the generic form of the related matrices η_a is

$$\eta_a = A_{n\times n} \otimes \sigma_a\,, \tag{6}$$

where $A_{n\times n}$ is an $n \times n$ matrix. The commutativity of matrices (6) leads to the nilpotency condition

$$A^2_{n\times n} = 0\,. \tag{7}$$

Thus, without any loss of generality, $A_{n\times n}$ may be expressed as a direct sum of 2×2 Jordan cells and zero matrices.

Because of (7) there exist only two different indecomposable representations of algebra hg(1, 3) defined on the spin $\frac{1}{2}$ carrier space:

$$S_a = \tfrac{1}{2}\,\sigma_a\,, \quad \eta_a = 0\,,$$

and

$$S_a = \frac{1}{2}\begin{pmatrix} \sigma_a & 0 \\ 0 & \sigma_a \end{pmatrix}, \quad \eta_a = \frac{1}{2}\begin{pmatrix} 0 & 0 \\ \sigma_a & 0 \end{pmatrix}. \tag{8}$$

The corresponding vectors from the representation space are two-component spinors $\varphi(\mathbf{x}, t)$ and four-component bispinors $\Psi = \begin{pmatrix} \varphi_1(\mathbf{x}, t) \\ \varphi_2(\mathbf{x}, t) \end{pmatrix}$, respectively, with two-component φ_1 and φ_2 [1]).

Consider the case $\tilde{s} = 1$, when spin s takes values 1 and 0. In basis $|s, m, \lambda_s\rangle$ matrices S_a are decomposed to direct sums of irreducible matrices of spin 1 and of spin zero, i.e., S_a and η_a can be written as

$$S_a = \begin{pmatrix} I_n \otimes s_a & \cdot \\ \cdot & 0_m \end{pmatrix}, \quad \eta_a = \begin{pmatrix} A \otimes s_a & B \otimes k_a \\ C \otimes k_a^\dagger & 0_m \end{pmatrix}, \tag{9}$$

where I_n is an $n \times n$ unit matrix and O_m an $m \times m$ zero matrix, A, B, and C are $n \times n$, $m \times n$, and $n \times m$ matrices, respectively, s_a are 3×3 spin-one matrices, whose elements are $(s_a)_{bc} = \mathrm{i}\varepsilon_{abc}$, and k_a are 1×3 matrices of the form

$$k_1 = \big(\mathrm{i},\, 0,\, 0\big)\,, \quad k_2 = \big(0,\, \mathrm{i},\, 0\big)\,, \quad k_3 = \big(0,\, 0,\, \mathrm{i}\big)\,. \tag{10}$$

[1]) for details see [6]

Matrices (9) satisfy relations (2) with arbitrary A, B, and C. Substituting (9) into (3) we come to the following equations:

$$A^2 + BC = 0\,, \tag{11}$$

$$CA = 0\,, \quad AB = 0\,. \tag{12}$$

Thus the problem of classification of finite-dimensional representations of algebra hg$(1,3)$ for $\tilde{s} = 1$ is reduced to the matrix problem (11). Notice that relations (11) are invariant w.r.t. to the following transformations:

$$A \to A' = \alpha W A W^{-1}\,, \quad B \to B' = \alpha W B V^{-1}\,, \quad C \to C' = \alpha V C W^{-1}\,, \tag{13}$$

where α is a complex non-zero multiplier, W and V are invertible matrices of dimension $n\times n$ and $m\times m$, respectively. Sets of matrices $\{A, B, C\}$ and $\{A', B', C'\}$ connected by relations (13) will be treated as equivalent.

The general solution of equations (11) up to the equivalence transformations (13) was found in [6]. Non-equivalent sets of indecomposable matrices S_a and η_a are labeled by three numbers n, m, and λ, where n and m take the values

$$-1 \leq n - m \leq 2\,, \quad n \leq 3 \tag{14}$$

and define dimensions of matrices $I_{n\times n}$ and $\mathbf{0}_{m\times m}$ in (9) and $\lambda = \operatorname{rank} B$, values of which are

$$\lambda = \begin{cases} 0 & \text{if} \;\; m = 0\,, \\ 1 & \text{if} \;\; m = 2 \;\; \text{or} \;\; n - m = 2\,, \\ 0,\,1 & \text{if} \;\; m = 1\,, \;\; n \neq 3\,. \end{cases} \tag{15}$$

The related matrices S_a and η_a are given in Table 1 of [6].

Thus we have found all indecomposable finite-dimensional realizations of the algebra hg$(1,3)$, which, when reduced to its compact subalgebra so(3), are decomposed to direct sums of irreducible representations corresponding to spins $\frac{1}{2}$, 1 and 0. It is possible to show [6] that for other values of spin the problems of complete description of indecomposable representations of hg$(1,3)$ are reduced to wild algebraic problems.

4 Finite transformations

Starting with the found representations of the Lie algebra of the homogeneous Galilei group and solving the related Lie equations, it is not difficult to derive the corresponding transformations from the group HG$(1,3)$. These transformations can be presented in the following compact form.

The carrier spaces of representations of the homogeneous Galilei group corresponding to the representations of algebra hg$(1,3)$, listed in Table 1 of [6], include three types of rotational scalars A, B, C and five types of vectors $\mathbf{R}$, $\mathbf{U}$, $\mathbf{W}$, $\mathbf{K}$, $\mathbf{N}$.

Their transformation laws with respect to Galilei boosts are given by

$$
\begin{aligned}
A &\to A' = A\,, \\
B &\to B' = B + \mathbf{v}\cdot\mathbf{R}\,, \\
C &\to C' = C + \mathbf{v}\cdot\mathbf{U} + \tfrac{1}{2}\,\mathbf{v}^2 A\,, \\
\mathbf{R} &\to \mathbf{R}' = \mathbf{R}\,, \\
\mathbf{U} &\to \mathbf{U}' = \mathbf{U} + \mathbf{v}A\,, \\
\mathbf{W} &\to \mathbf{W}' = \mathbf{W} + \mathbf{v}\times\mathbf{R}\,, \\
\mathbf{K} &\to \mathbf{K}' = \mathbf{K} + \mathbf{v}\times\mathbf{R} + \mathbf{v}A\,, \\
\mathbf{N} &\to \mathbf{N}' = \mathbf{N} + \mathbf{v}\times\mathbf{W} + \mathbf{v}B + \mathbf{v}\big(\mathbf{v}\cdot\mathbf{R}\big) - \tfrac{1}{2}\,\mathbf{v}^2\mathbf{R}\,,
\end{aligned}
\tag{16}
$$

where $\mathbf{v} = (v_1, v_2, v_3)$ are transformation parameters and $\mathbf{v}\cdot\mathbf{R}$ and $\mathbf{v}\times\mathbf{R}$ are scalar and vector products of vectors $\mathbf{v}$ and $\mathbf{R}$, respectively.

The spaces of indecomposable representations of group $\mathrm{HG}(1,3)$ include such sets of scalars A, B, C and vectors $\mathbf{R}$, $\mathbf{U}$, $\mathbf{W}$, $\mathbf{K}$, $\mathbf{N}$, which are decoupled w.r.t. the transformations (16). There exist exactly ten such sets:

$$
\begin{array}{lllll}
1.\ \{A\}\,, & 2.\ \{\mathbf{R}\}\,, & 3.\ \{B,\mathbf{R}\}\,, & 4.\ \{A,\mathbf{U}\}\,, & 5.\ \{A,\mathbf{U},C\}\,, \\
6.\ \{\mathbf{W},\mathbf{R}\}\,, & 7.\ \{\mathbf{R},\mathbf{W},B\}\,, & 8.\ \{A,\mathbf{K},\mathbf{R}\}\,, & 9.\ \{A,B,\mathbf{K},\mathbf{R}\}\,, & 10.\ \{A,\mathbf{N},\mathbf{W},\mathbf{R}\}\,,
\end{array}
$$

which are arranged in the same order as the corresponding representations of $\mathrm{hg}(1,3)$ in Table 1 of [6], i.e., $\{A\}$ belongs to the space of representation $D(0,0,0)$, $\{\mathbf{R}\}$ belongs to the space of representation $D(1,0,0)$, etc.

5 Contractions of representations of the Lorentz algebra

It is well known that the Galilei algebra and (some of) its representations can be obtained from the Poincaré algebra and from its appropriate representations by a limiting procedure called "contraction" [7].

In the simplest case, *contraction* is defined as a limiting procedure, by which an N-dimensional Lie algebra $\mathcal{L}$ is transformed into another, non-isomorphic N-dimensional Lie algebra $\mathcal{L}'$. The commutation relations of the *contracted Lie algebra* $\mathcal{L}'$ are given by:

$$
[x,y]' \equiv \lim_{\varepsilon\to\varepsilon_0} W_\varepsilon^{-1}\big[W_\varepsilon(x), W_\varepsilon(y)\big]\,,
\tag{17}
$$

where $[.\,,.]$ denotes the binary operation $\mathcal{L}$, $W_\varepsilon \in \mathrm{GL}(N,k)$ is a non-singular linear transformation of $\mathcal{L}$ with ε_0 being a singularity point of its inverse W_ε^{-1}.

There is a simple Inönü–Wigner contraction procedure connecting the Lie algebra $\mathrm{so}(1,3)$ of the Lorentz group with the algebra $\mathrm{hg}(1,3)$. The related transformation W does not change basis elements of $\mathrm{so}(1,3)$ forming its subalgebra $\mathrm{so}(3)$, while the remaining basis elements are multiplied by a small parameter ε, which tends to zero [7]. However, the corresponding contraction of *representations* is by no means simple, since in order to obtain indecomposable representations of $\mathrm{hg}(1,3)$ it is necessary to begin with completely reducible representations of the Lie algebra of the Lorentz group.

Here we specify representations of the Lorentz group, which via specified contractions go to desired realizations of the homogeneous Galilei group.

Let $S_{\mu\nu}$, μ, $\nu = 0, 1, 2, 3$ be matrices realizing a representation of the algebra so$(1, 3)$, i.e., satisfying the relations

$$\big[S_{\mu\nu}, S_{\lambda\sigma}\big] = \mathrm{i}\big(g_{\mu\lambda}S_{\nu\sigma} + g_{\nu\sigma}S_{\mu\lambda} - g_{\nu\lambda}S_{\mu\sigma} - g_{\mu\sigma}S_{\nu\lambda}\big) \tag{18}$$

with $g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$. The Inönü–Wigner contraction procedure consists of a changing basis of algebra so$(1, 3)$ to another, new one by

$$S_{ab} \to S_{ab}\,, \quad S_{0a} \to \varepsilon S_{0a}$$

and simultaneously transforming all basis elements $S_{\mu\nu} \to S'_{\mu\nu} = US_{\mu\nu}U^{-1}$ with a matrix U depending on ε. Moreover, U should depend on ε in a tricky way, so that all the transformed generators $S'_{\mu\nu}$ are kept non-trivial when $\varepsilon \to 0$ [7].

We suppose that representations obtained by the contraction are indecomposable representations $D(n, m, \lambda)$ presented in Table 1 of [6]. To construct representations of the Lorentz algebra, which can be contracted to representations $D(n, m, \lambda)$, we state the following lemma [6].

Lemma 1. *Let $\{S_a, \eta_a\}$ be an indecomposable set of matrices realizing one of representations $D(1, 1, 0)$, $D(1, 1, 1)$, $D(1, 2, 1)$, $D(2, 0, 0)$ or $D(3, 1, 1)$ presented in Table 1 of* [6]. *Then matrices*

$$S_{ab} = \varepsilon_{abs}S_c\,, \quad S_{0a} = \nu(\eta_a - \eta_a^\dagger)\,, \tag{19}$$

where $\nu = 1$ for representations $D(1, 1, 0)$, $D(1, 1, 1)$, $D(2, 0, 0)$ and $\nu = 1/\sqrt{2}$ for representations $D(1, 2, 1)$, $D(3, 1, 1)$, form a basis of the Lie algebra of the Lorentz group.

For representations $D(1, 0, 0)$, $\big(D(2, 1, 1), D(2, 1, 0)\big)$, and $D(2, 2, 1)$ the related matrices (19) do not form a basis of a Lie algebra. Nevertheless it is possible to find the corresponding generators of the Lorentz algebra starting with its known representation $D(1, 0)$ realized by matrices of dimension 3×3, representation $D(\frac{1}{2}, \frac{1}{2}) \oplus D(1, 0)$ realized by matrices of dimension 7×7 and representation $D(\frac{1}{2}, \frac{1}{2})\oplus D(1, 0)\oplus D(0, 0)$ realized by matrices of dimension 8×8, correspondingly. We choose these representations in the following forms:

$$S_{ab} = \varepsilon_{abc}s_c\,, \quad S_{0a} = \mathrm{i}s_a\,, \tag{20}$$

$$S_{ab} = \varepsilon_{abc}\begin{pmatrix} s_c & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times1} \\ \mathbf{0}_{3\times3} & s_c & \mathbf{0}_{3\times1} \\ \mathbf{0}_{1\times3} & \mathbf{0}_{1\times3} & 0 \end{pmatrix}, \quad S_{0a} = \frac{1}{2}\begin{pmatrix} \mathrm{i}s_a & -s_a & \mathrm{i}\sqrt{2}k_a^\dagger \\ s_a & \mathrm{i}s_a & -\sqrt{2}k_a^\dagger \\ \mathrm{i}\sqrt{2}k_a & \sqrt{2}k_a & 0 \end{pmatrix}, \tag{21}$$

and

$$S_{ab} = \varepsilon_{abc}\begin{pmatrix} s_c & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times1} & \mathbf{0}_{3\times1} \\ \mathbf{0}_{3\times3} & s_c & \mathbf{0}_{3\times1} & \mathbf{0}_{3\times1} \\ \mathbf{0}_{1\times3} & \mathbf{0}_{1\times3} & 0 & 0 \\ \mathbf{0}_{1\times3} & \mathbf{0}_{1\times3} & 0 & 0 \end{pmatrix}, \quad S_{0a} = \frac{1}{2}\begin{pmatrix} \mathrm{i}s_a & -s_a & \mathrm{i}k_a^\dagger & k_a^\dagger \\ s_a & \mathrm{i}s_a & -k_a^\dagger & \mathrm{i}k_a^\dagger \\ \mathrm{i}k_a & k_a & 0 & 0 \\ -k_a & \mathrm{i}k_a & 0 & 0 \end{pmatrix}. \tag{22}$$

Starting with realizations of $\mathrm{so}(1,3)$ described in Lemma and Eqs. (20)–(22) and applying the contraction procedures, it is possible to obtain all representations of algebra $\mathrm{hg}(1,3)$ present in Table 1 of [6]. These contractions are specified explicitly in Table 2 of [6].

Thus using the contraction procedure we have obtained the indecomposable vector and scalar representations of the Lie algebra of the homogeneous Galilei group starting with appropriate finite-dimensional representations of algebra $\mathrm{so}(1,3)$. Notice that to obtain *indecomposable* representations of algebra $\mathrm{hg}(1,3)$ found in Section 4, we had to use *completely reducible* representations of the Lorentz algebra.

6 Functional invariants

In this section we find all functional invariants for each representation described in Sections 3 and 4 applying the moving frame approach [8]. It is especially convenient in our case, when the finite group transformations are already known and, moreover, transformed functions (16) depends algebraically on the transformation parameters.

We shall construct the invariants in two steps. First we algebraically exclude transformation parameters $\mathbf{v}$ from transformation formulae (16) and obtain invariants for the Galilei boost transformations. Then, using standard tensor calculus, we easily construct invariants for both the boost and the rotation transformations.

Consider all indecomposable representations presented in Section 4, one after the other.

Let the representation space include vector $\mathbf{U}$. Then it is possible to exclude the transformation parameters $\mathbf{v}$,

$$\mathbf{v} = \frac{1}{A}\left(\mathbf{U}' - \mathbf{U}\right). \tag{23}$$

Table 1.

No	Representation	Invariants
1	$D(0,0,0)$	A
2	$D(1,0,0)$	$\mathbf{R}^2$
3	$D(1,1,0)$	$\mathbf{R}^2$
4	$D(1,1,1)$	A
5	$D(1,2,1)$	$2AB - \mathbf{R}^2$
6	$D(2,0,0)$	$\mathbf{R}^2,\ \mathbf{W}\cdot\mathbf{R}$
7	$D(2,1,0)$	$\mathbf{R}^2,\ \mathbf{W}\cdot\mathbf{R}$
8	$D(2,1,1)$	$A,\ \mathbf{R}^2$
9	$D(2,2,1)$	$A,\ \mathbf{R}^2,\ AB - \mathbf{K}\cdot\mathbf{R}$
10	$D(3,1,2)$	$\mathbf{R}^2,\ \mathbf{W}\cdot\mathbf{R},\ B^2 - \mathbf{W}^2 - 2\mathbf{N}\cdot\mathbf{R}$, $2\mathbf{R}^2\mathbf{N}^2 + (\mathbf{W}^2 + B^2)^2 + 2\mathbf{R}\cdot\mathbf{N}(\mathbf{W}^2 - \mathbf{R}^2)$ $- 4(\mathbf{R}\cdot\mathbf{W})(\mathbf{N}\cdot\mathbf{W}) - 4B(\mathbf{N}\cdot\mathbf{R}\times\mathbf{W})$

If the representation space includes vectors $\mathbf{W}$ and scalar B, we again can express $\mathbf{v}$ via transformed and untransformed functions and obtain

$$\mathbf{v} = \frac{1}{\mathbf{R}'^2}\left(\mathbf{R}' \times \mathbf{W}' + B'\mathbf{R}'\right) - \frac{1}{\mathbf{R}^2}\left(\mathbf{R} \times \mathbf{W} + B\mathbf{R}\right). \qquad (24)$$

Using (23) and (24), we find invariants w.r.t. Galilean boosts and then calculate invariants w.r.t. rotations depending on these scalars. In this way we can get the complete list of invariants for all indecomposable representations presented in Section 4. This list is given in the Table 1.

It is possible to show that all the other invariants w.r.t. the Galilei group are functions of the ones presented in the Table 1.

7 Discussion

In this paper we completely describe indecomposable representations of the homogeneous Galilei group which, being reduced to the maximal compact subgroup SO(3), are decomposed into the direct sum of spin 0, $\frac{1}{2}$, and 1 representations. These results can be used to construct various physical models satisfying the Galilei relativity principle.

Let us note that the homogeneous Galilei group is locally isomorphic to the Euclidean group E(3) in three-dimensional Riemanian space. Thus we have effectively described a class of finite-dimensional representations of E(3).

Let us note that it seems to be impossible to generalize the obtained results to the cases of Galilei and Euclid groups in spaces with more dimensions than (1+3) and 3. Indeed, even for Galilei group in (1+4)-dimensional space the problem of description of finite-dimensional representations is wild.

The obtained results can be applied to derive Galilei-invariant equations for spinor and vector particles. This work is in progress.

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