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MATRIX METHODS  
FOR THE ANALYSIS  
AND SYNTHESIS  
OF DYNAMICAL  
SYSTEMS

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*PROJECT*  
*«SCIENTIFIC BOOK»*

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The book outlines constructive methods for the analysis and synthesis of dynamical systems based on the application of matrix equations and inequalities. The generalizations of the Lyapunov equation are presented in the framework of stability and spectrum localization theory of linear systems. Classical methods for motion stability analysis, modern methods for robust stabilization and optimization of dynamical systems, as well as new approaches to solving generalized  $H_\infty$ -control problems for continuous and discrete time systems with controllable and observable outputs are presented. Algorithms for estimating and minimizing the weighted damping level of bounded perturbations in standard and descriptor control systems are proposed. The stability theory of positive and monotone dynamical systems is developed.

The book is intended for scientists, engineers, PhD students and senior students of the corresponding specialties.