Wronski — Schubert Calculus

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I will report on a joint work with Letterio Gatto (Politecnico di Torino, Italy).

The ring of **generalized Wronskians** is a natural and useful extension of the classical notion of the Wronskian of smooth functions in one variable. This ring contains all derivatives of the Wronskian. In the talk we discuss the relationship with the intersection theory on Grassmann varieties, ruled by Schubert Calculus.

We show that the generalized Wronskians associated to a fundamental system of solutions of a linear ordinary differential equation with constant coefficients enjoy Giambelli's like formula: The ratio between a generalized Wronskian and the Wronskian is a Schur polynomial in certain polynomial expressions of the coefficients of a given differential equation. This generalizes the classical theorem due to Abel and Liouville claiming that the Wronskian and its first derivative are proportional. It turns out that Schubert cycles can be interpreted as generalized Wronskians of a fundamental system of solutions of a linear ODE with coefficients taken in the intersection ring of the Grassmannian.

To our knowledge, generalized Wronskians appeared first in a work by F. H. Schmidt of 1939 [Math. Z., v.45, 62—74] in his studies of Weierstrass points on curves. Recently, they appeared with similar motivations in works by L.Gatto, F.Ponsa and C.Towse, and in connection with number theory in works by G.Anderson and by A.Milas&E.Mortenson&K.Onoper.