A NEW TYPE OF q-DIFFERENTIAL GEOMETRY

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We introduce q-analogues of the basic concepts in nineteenth century differential geometry. Under the assumption of q-real numbers, we generalize the previously introduced notion of 3×3 q-determinant to τ operating on both the second and third row, but with different indices as well as to other forms. The q-scalar product is generalized in an analogous way. This brings us to define the corresponding q-metric. It turns out that, in this context, the most common orthogonal coordinates have similar q-aliases, which enable q-analogues of coordinate surfaces, like the q-sphere, etc. The concept of q-Gaussian curvature is introduced together with q-minimal surfaces. The concept q-differential in vector form is introduced to prepare for the first fundamental form and the q-metric. After the introduction of q-Christoffel symbols, we consider three different aspects of q-geodesics, the second order q-difference equation, the coordinate surfaces and the q-exponential function. The five-dimensional q-de Sitter spacetime corresponds to a sphere in q-Minkowski spacetime. The matrix pseudogroup $SL_q(2;\mathbb{R})$ as function of four coordinates is a kind of three-dimensional q-hyperboloid in $\mathbb{R}^4$.

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