Rotational symmetry and degeneracy in the cotangent perturbed rigid rotator and Romanovski polynomials

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We raise the question on the degeneracy phenomenon in the spectra of some exactly solvable two-terms potentials, a phenomenon which presents itself unexpected upon relating the potentials to geodesic motions on appropriate curved surfaces. The case is made on the example of the trigonometric Rosen-Morse potential, $\mathcal{V} = l(l+1)\csc^2\eta - 2b\cot\eta$, considered as an angular function on the S^2 sphere, whose isometry group is SO(3). From this perspective, the first term naturally emerges as part of the Casimir invariant, $\mathbf{L}^2$, of the so(3) algebra, whose eigenvalue problem, $\mathbf{L}^2 Y_l^m(\theta,\varphi) = l(l+1)Y_l^m(\theta,\varphi)$, determines the spectrum of the free geodesic motion in terms of the spherical harmonics, $Y_l^m(\theta,\varphi) = P_l^m(\cos\theta)e^{im\varphi}$, while the second term provokes a perturbance. The unexpected property of the $\cot\theta$ perturbance is that despite its non-commutativity with $\mathbf{L}^2$, it still conserves the $(2l+1)$-fold degeneracy of the free geodesic motion. In order to clarify how such can occur, we first construct simple decompositions of the solutions of the trigonometric Rosen-Morse potential, expressed in terms of non-classical Romanovski polynomials, in the basis of exponentially scaled spherical harmonics. Next we demonstrate that by virtue of specific recurrence relations among these very harmonics, the $\left(\mathbf{L}^2 - 2b\cot\eta\right)$, and $\mathbf{L}^2$ eigenvalue problems appear equivalent up to a non-unitary scaling similarity transformation. As long as any similarity transformation of the algebra preserves dimensionality and labellings of the representation bases, the $\cot\eta$ potential preserves the degeneracy of the states, though it breaks the rotational SO(3) group symmetry at the level of the representation (wave) functions. In this fashion, one encounters a mode of symmetry breaking at the level of the representation functions, opaqued by degeneracy patterns in the spectra. Consequences for physical processes are briefly discussed.