DOI 10.1007/s11253-014-0993-9
Ukrainian Mathematical Journal, Vol. 66, No. 7, December, 2014 (Ukrainian Original Vol. 66, No. 7, July, 2014)

REPRESENTATION TYPE OF NODAL ALGEBRAS OF TYPE D

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UDC 512.552.8

We establish the representation types (finite, tame, or wild) of nodal algebras of type D.

The present paper is a continuation of paper [1] in which nodal algebras are introduced and the representation types (finite, tame, or wild) of nodal algebras of type A, i.e., algebras obtained by blowing up and gluing of quivers of type A (or of type $\tilde{\mathrm{A}}$) are established. In the present paper, we determine the representation types of nodal algebras of type D, i.e., algebras obtained by blowing up and gluing of quivers of type D (or $\tilde{\mathrm{D}}$) that are not nodal algebras of type A.

1. Nodal Algebras

We fix an algebraically closed field $\mathbf{k}$ and consider only finite-dimensional $\mathbf{k}$-algebras. We now recall the definition and structure of nodal algebras [1, 2].

Definition 1.1. *An algebra A is called nodal if there exists a hereditary algebra $H \supset A$ such that:*

(1) $\operatorname{rad} A = \operatorname{rad} H$;

(2) $\operatorname{length}_A(H \otimes_A U) \leq 2$ *for any simple left A-module U.*

We say that the nodal algebra A is connected with the hereditary algebra H.

Recall that an algebra A is called *base* [3] if its quotient algebra $\bar{A} = A/\operatorname{rad} A$ is isomorphic to the direct product of fields. Since we consider algebras over an algebraically closed field $\mathbf{k}$, in the analyzed case, we have $A/\operatorname{rad} A \simeq \mathbf{k}^m$ for some m. By the Morita theorem [3], the algebra and its base algebra have the same representation type. In [1], we show that if an algebra A is nodal and connected with the hereditary algebra H, then its base algebra is nodal and connected with a hereditary algebra Morita-equivalent to H. For this reason, in what follows, we consider only base nodal algebras.

We now recall the structure that gives all base nodal algebras [1, 2]. We define *nodal data* as a collection including

(1) a quiver Q;

(2) a binary symmetric relation $\sim$ on the set Q_0 (vertices of the quiver Q) such that, for each vertex $i \in Q_0$, there exists at most one vertex $j \in Q_0$ for which $i \sim j$.

These data are used construct a base nodal algebra $A(Q, \sim)$ in the following way:

1. Consider a hereditary algebra H with a quiver Q and the multiplicities of the vertices

$$m_j = \begin{cases} 1 & \text{if} \quad j \nsim j, \\ 2 & \text{if} \quad j \sim j. \end{cases}$$

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Translated from Ukrains'kyi Matematychnyi Zhurnal, Vol. 66, No. 7, pp. 930–938, July, 2014. Original article submitted September 18, 2013.

2. In a quotient algebra

$$\bar{H} = H/\operatorname{rad} H = \prod_{j=1}^{s} \operatorname{Mat}(m_j, \mathbf{k}),$$

we consider a subalgebra $\bar{A}$ formed by collections $(a_1, \ldots, a_s)$ such that

$a_j = a_k$ if $j \sim k$ and $k \neq j$; in this case, we say that A is obtained from H *by gluing the vertices* j and k of the quiver Q;

a_j is a diagonal matrix if $j \sim j$; in this case, we say that A is obtained from H *by blowing up the vertex* j of the quiver Q.

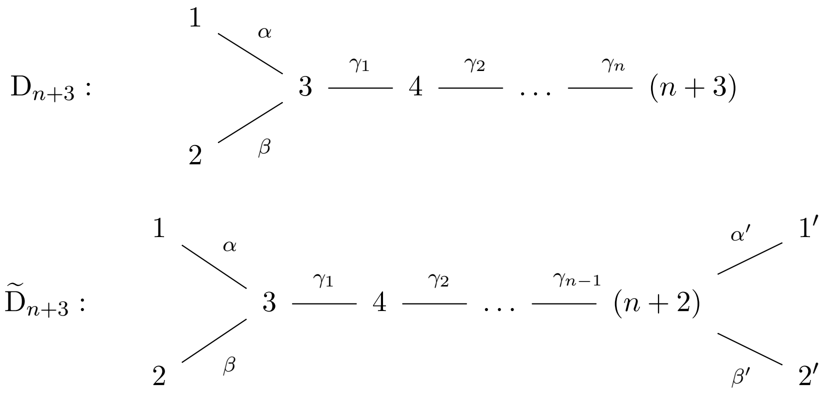
3. Consider a subalgebra $A = A(Q, \sim) \subset H$ obtained as the preimage of the subalgebra $\bar{A} \subset \bar{H}$; by construction, this subalgebra is a base algebra.

In [1, 2], it is proved that each base nodal algebra A is isomorphic to an algebra obtained from the base hereditary algebra H with quiver Q as a result of a certain sequence of operations of gluing and blowing up of vertices of this quiver.

Recall that the gluing of vertices i and j of the quiver Q is called *inessential* if there are no arrows entering the vertex i (resp., leaving the vertex i) and arrows leaving the vertex j (resp., entering the vertex j). It is known that these gluings do not affect the representation type of the algebra A [1]

2. Nodal Algebras of Type D

If Q is a quiver of type A (or $\tilde{\mathrm{A}}$), then we say that the nodal algebra A is *a nodal algebra of type* A. In this case, the representation type of the algebra A (finite, tame, or wild) is determined in [1]. We are interested in the case where Q is a quiver of type D (or $\widetilde{\mathrm{D}}$):



(2.1)

with arbitrary orientation of the edges. In what follows, we use the notation of vertices and arrows presented in (2.1).

By Q', we denote a quiver obtained from Q by removing vertex 2 and the edge β. If Q is a quiver of type D, then Q' is a quiver of type A; if Q is a quiver of type $\widetilde{\mathrm{D}}$, then $Q'$is a quiver of type D.

Remark 2.1. Assume that vertices 1 and 2 do not participate in gluings. If both arrows α and β originate (or end) at the vertex 3, then the algebra A can be obtained from the quiver Q' by the same of operations of gluing and blowing up supplemented with the operation of blowing up of vertex 1. Assume that one of these arrows originates at vertex 3 and the other arrow terminates at this vertex. If α (or β) does not belong to the relations of the algebra A, then we can apply the operation of reflection from[4] to vertex 1 (resp., to vertex 2) in order to get a representation of the algebra that differs from A solely by the orientation of the arrow α (reap., β). It is easy to see that, for any operations of gluing in which vertices 1 and 2 do not participate and any operations of blowing up of vertices other than3, at least one of the arrows (α or β) is not contained in the relations. Hence, this case is reduced to the previous case. Clearly, the same is true for vertices $1'$ and $2'$ in the case of a quiver of type $\widetilde{\mathrm{D}}$.

This remark enables us to introduce the following definition:

Definition 2.1. *A nodal algebra A is called a nodal algebra of type D if, in the corresponding nodal data, the quiver Q has the type D or $\widetilde{\mathrm{D}}$ and, in addition, either one of vertices 1 and 2 takes part in gluing or vertex 3 is blown up. In the case of a quiver of type $\widetilde{\mathrm{D}}$, it is additionally supposed that either one of vertices $1'$ and $2'$ takes part in gluing or vertex $(n+2)$ is blown up.*

The theorems presented in what follows describe the representation types of nodal algebras of type D. Moreover, the direction of arrows omitted in the diagrams is arbitrary and does not affect the representation type.

Theorem 2.1. *Let a nodal algebra A be isomorphic or antiisomorphic to an algebra obtained from a quiver of type D by certain inessential gluings and one of the following operations:*

(1) gluing of vertices 1 and 3 in the quiver

$$\begin{tikzcd}
1 \arrow[dr, "\alpha"] & & & & \\
& 3 \arrow[r, "\gamma_1"] & 4 \arrow[r, dash, "\gamma_2"] & \dots \arrow[r, dash, "\gamma_n"] & (n+3) \\
2 \arrow[ur, "\beta"'] & & & &
\end{tikzcd} \tag{2.2}$$

for $n \leq 2$;

(2) gluing of vertices 1 and 3 in the quiver

$$\begin{tikzcd}
1 \arrow[dr, "\alpha"] & & & & \\
& 3 \arrow[dl, "\beta"] & 4 \arrow[l, "\gamma_1"'] \arrow[r, dash, "\gamma_2"] & \dots \arrow[r, dash, "\gamma_n"] & (n+3) \\
2 & & & &
\end{tikzcd} \tag{2.3}$$

for $n \leq 4$;

(3) gluing of vertices 1 and $(n+2)$ in the quiver

$$\begin{tikzcd}
1 \arrow[dr, "\alpha"] & & & & \\
& 3 \arrow[r, dash, "\gamma_1"] & \dots \arrow[r, "\gamma_{n-1}"] & (n+2) \arrow[r, "\gamma_n"] & (n+3) \\
2 \arrow[ur, dash, "\beta"'] & & & &
\end{tikzcd} \tag{2.4}$$

for $2 \leq n \leq 4$.

Then A has a finite representation type.

Theorem 2.2. *Let a nodal algebra A be isomorphic or antiisomorphic to an algebra obtained from a quiver of type D or $\widetilde{\mathrm{D}}$ by certain inessential gluings and one of the following operations:*

(1) gluing of vertices 1 and 3 in quiver (2.3) for $n = 5$;

(2) gluing of vertices 1 and 3 in the quiver

$$\begin{tikzcd}
1 \arrow[dr, "\alpha"] & & \\
& 3 \arrow[r, "\gamma_1"] \arrow[dl, "\beta"] & 4\,; \\
2 & &
\end{tikzcd}$$

(3) gluing of vertices 1 and 4 in the quiver

$$\begin{tikzcd}
1 \arrow[dr, "\alpha"] & & & & \\
& 3 \arrow[r, "\gamma_1"] \arrow[dl, dash, "\beta"] & 4 \arrow[r, "\gamma_2"] & 5 \arrow[r, dash, "\gamma_3"] & 6\,; \\
2 & & & &
\end{tikzcd}$$

(4) gluing of vertices 1 and $(n+2)$ in quiver (2.4) for $n = 5$;

(5) blowing up of vertex 3 in the quiver

$$\begin{tikzcd}
1 \arrow[dr, "\alpha"] & & \\
& 3 \arrow[dl, "\beta"] & 4\,; \arrow[l, "\gamma"'] \\
2 & &
\end{tikzcd} \tag{2.5}$$

(6) blowing up of vertex 3 in the quiver

$$\begin{tikzcd}
1 \arrow[dr, "\alpha"] & & 4 \arrow[dl, "\gamma"'] \\
& 3 \arrow[dl, "\beta"] \arrow[dr, "\delta"] & \\
2 & & 5
\end{tikzcd} . \tag{2.6}$$

Then A is a time algebra of infinite representation type.

Theorem 2.3. *If a nodal algebra A of type D is neither isomorphic nor antiisomorphic to an algebra from the families of algebras described in Theorems 1 and 2, then it is wild.*

3. Proof of the Theorems

We simultaneously prove Theorems 2.1–2.3. Moreover, we consider special cases of gluing and blowing up. To within the isomorphism or antiisomorphism, we can assume that the arrow α is directed from vertex 1 to vertex 3, furthermore, either vertex 1 takes part in gluing or blowing up of vertex 3 occurs.

Case 3.1. Gluing of vertices 1 and 3 in quiver (2.2).

We obtain a quiver of the form

$$\begin{tikzcd}
2 \arrow[r, "\beta"'] & 3 \arrow[loop above, "\alpha"] \arrow[r, "\gamma_1"] & 4 \arrow[r, dash, "\gamma_2"] & \dots \arrow[r, dash, "\gamma_n"] & (n+3)
\end{tikzcd}$$

$$\text{with relations} \quad \alpha^2 = 0, \quad \alpha\beta = 0.$$

As a result of the reduction

$$\alpha \rightsquigarrow \begin{pmatrix} 0 & 0 & I \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

the rows β are split into three parts, furthermore, by virtue of the condition

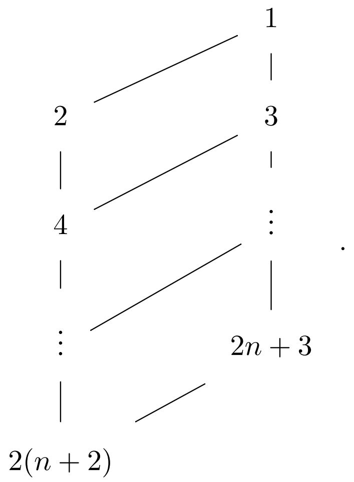
$$\alpha\beta = 0,$$

the last row is zero. We reduce β to the form

$$\beta \rightsquigarrow \left(\begin{array}{ccc} 0 & I & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & I \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right).$$

Then γ_1 is split into six columns. In this case, columns 1 and 2 and columns 5 and 6 *are connected*, i.e., they are subjected to the same transformations. As a result of the reduction of arrows $\gamma_2, \dots, \gamma_n$, the rows of the matrix γ_1 are split into n parts, furthermore, they are linearly ordered. As a result of the reduction of the first two columns of this matrix, each column 5 and 6 is split into $n+1$ parts. For columns 3–6, we obtain $2n+4$ nonzero columns.

The addition of these columns is specified by a partially ordered set S of the form



It follows from [5] that our problem is equivalent to the problem of representation of a partially ordered set that is a cardinal sum S and a set that is linearly ordered and has $n-1$ elements. It follows from [5, 6] that this problem is finite for $n \leq 2$ and wild for $n > 2$. Hence, the same statement is true for the algebra A. This gives the first assertion of Theorem 2.1.

Any additional essential gluing or blowing up additionally divides the matrix β or γ_1, and the problem becomes wild. As an example, consider the case of additional gluing of vertices 4 and 5 under condition that the arrow γ_2 is directed to vertex 5 (otherwise, this gluing is inessential). The resulting quiver with relations contains the subquiver

$$\begin{tikzcd}
2 \arrow[r, "\beta"'] & 3 \arrow[loop above, "\alpha"] \arrow[r, "\gamma_1"] & 4 \arrow[loop above, "\gamma_2"]
\end{tikzcd}$$

$$\text{with relations} \quad \alpha^2 = 0, \quad \alpha\beta = 0, \quad \gamma_2^2 = 0.$$

As a result of the reduction of α and γ_2 to the form

$$\begin{pmatrix} 0 & 0 & I \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

the matrices β and γ_1 are reduced to the form

$$\beta = \begin{pmatrix} B_1 \\ B_2 \\ 0 \end{pmatrix}\Bigg\uparrow \qquad \gamma_1 = \overrightarrow{\begin{pmatrix} G_{11}^* & G_{12} & G_{13}^* \\ G_{21} & G_{22} & G_{23} \\ G_{31}^* & G_{32} & G_{33}^* \end{pmatrix}}\Bigg\uparrow .$$

The arrows indicate the direction of transformations, furthermore, transformations of rows in the matrix β and columns in the matrix γ_1 are *contragradient*[1] and matrices with asterisks have common transformations of rows and columns. Consider representations for which $B_1 = 0,\ B_2 = I$, the matrix γ_1 does not have the second horizontal strip $\left(\text{i.e., } \gamma_2 = \begin{pmatrix} 0 & I \\ 0 & 0 \end{pmatrix}\right)$, $G_{33} = I$, and the matrices G_{31}, G_{32}, and G_{13} are null matrices. It is easy to see that this representation is isomorphic to a similar representation with matrices G'_{11} and G'_{12} in the corresponding places if and only if there exist nondegenerate matrices C_1 and C_2 such that

$$G'_{11} = C_1 G_{11} C_1^{-1}$$

and

$$G'_{12} = C_1 G_{12} C_2.$$

In other words, the matrices G_{11} and G_{12} define the representation of a wild quiver

$$\begin{tikzcd}
\bullet \arrow[loop left] \arrow[r] & \bullet
\end{tikzcd} .$$

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[1] This means that if the matrix β is multiplied from the left by a nondegenerate matrix C, then the matrix γ_1 is multiplied from the right by C^{-1}.

Hence, the algebra A is wild. For other additional operations of gluing or blowing up, the proof of wildness is similar (and, as a rule, even simpler). This proves Theorem 2.3 in the case of gluing of vertices 1 and 3 in quiver (2.2).

Case 3.2. Gluing of vertices 1 and 3 in quiver (2.3).

As a result, we obtain the quiver

$$\begin{tikzcd}
2 & 3 \arrow[l, "\beta"'] \arrow[loop above, "\alpha"] & 4 \arrow[l, "\gamma_1"'] \arrow[r, dash, "\gamma_2"] & \dots \arrow[r, dash, "\gamma_n"] & (n+3)
\end{tikzcd}$$

$$\text{with relations} \quad \alpha^2 = 0, \quad \alpha\gamma_1 = 0.$$

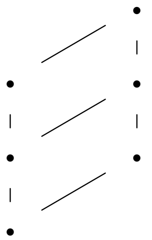
As a result of the reduction

$$\alpha \rightsquigarrow \begin{pmatrix} 0 & 0 & I \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

the columns β and, respectively, the rows γ_1 are split into three parts, furthermore, by virtue of the condition $\alpha\gamma_1 = 0$, the last part in γ_1 is zero. By performing transformations that do not change the form of the matrix α, we reduce the matrix β to the form

$$\beta \rightsquigarrow \left(\begin{array}{cccc|cc|cccc} 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right).$$

Correspondingly, the matrix γ_1 is split into ten horizontal parts but only the first six parts are nonzero. As a result of the reduction of the matrices $\gamma_2, \dots, \gamma_n$, the matrix γ_1 is additionally split into n vertical parts, i.e., γ_1 contains six nonzero rows and n columns. It is easy to see that addition of rows is defined by a partially order set S of the form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and addition of columns is defined by a linearly ordered set that consists of n elements. Thus, according to [5], our problem is equivalent to the problem of representation of a partially ordered set that is a cardinal sum S and a set that is linearly ordered and has $n-1$ elements. According to [5, 6], for $n \leq 4$, this problem has a finite type, for $n=5$, it is tame (of infinite type), and for $n>5$, it is wild. Hence, the same statements are true for the algebra A. This gives the second assertion of Theorem 2.1 and the first assertion of Theorem 2.2. Any additional essential gluing or blowing up gives a wild matrix problem. This proves Theorem 2.3 in the case of gluing of vertices 1 and 3 in quiver (2.3).

Case 3.3. Gluing of vertices 1 and 3 in the quiver

$$\begin{tikzcd}
1 \arrow[dr, "\alpha"] & & & & \\
& 3 \arrow[r, "\gamma_1"] \arrow[dl, "\beta"] & 4 \arrow[r, dash, "\gamma_2"] & \dots \arrow[r, dash, "\gamma_n"] & (n+3)\,. \\
2 & & & &
\end{tikzcd} \qquad (3.1)$$

We obtain the quiver

$$\begin{tikzcd}
2 & 3 \arrow[l, "\beta"'] \arrow[loop above, "\alpha"] \arrow[r, "\gamma_1"'] & 4 \arrow[r, dash, "\gamma_2"] & \dots \arrow[r, dash, "\gamma_n"] & (n+3)
\end{tikzcd}$$

with relation $\alpha^2 = 0$. We reduce the matrix α in the same way as in the previous cases. The columns of the matrices β and γ_1 are split into three parts that can be added from left to right, furthermore, transformations of the first and third parts must the same. As a result of the reduction of the matrices $\gamma_2, \ldots, \gamma_n$, the rows of the matrix γ_1 are split into n parts in which transformations are defined by a linearly ordered set. If $n=1$, then we obtain the problem of representation of a bundle of chains

$$\mathfrak{E} = \{\, e \,\}, \qquad \mathfrak{F} = \{\, f_1 < f_2 < f_3 \,\}$$

with relation $\sim$ such that $e \sim e$ and $f_1 \sim f_3$[2]. This problem is tame (of infinite type). The same is true for the algebra A. If $n>1$, then we consider representations in which $\gamma_3 = \ldots = \gamma_n = 0$, the second vertical part of the matrices β and γ_1 is absent $\left(\text{i.e., } \alpha = \begin{pmatrix} 0 & I \\ 0 & 0 \end{pmatrix}\right)$, and the first and third vertical parts are reduced to the form

$$\left(\begin{array}{cc|cc}
0 & 0 & G_1 & G_2 \\
I & 0 & 0 & 0 \\
\hline\hline
0 & 0 & G_3 & G_4 \\
\hline
0 & 0 & G_5 & G_6 \\
0 & I & 0 & 0
\end{array}\right).$$

Here, the double horizontal line is the line of separation between the matrices β and γ_1 and the single horizontal line corresponds to the separation between the parts of the matrix γ_1 formed as a result of the reduction of

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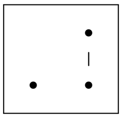
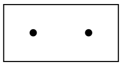
[2] We use the definition of a bundle of chains in [7]. In [8], for the same problems, the author used the term "*bundle of semichains*" and another coding.

the matrix γ_2. It is easy to see that the matrices G_i form representations of a pair of partially ordered sets

$S =$  and $T =$  .

It follows from [9] that this problem and, hence, the algebra A are wild. This proves the second assertion of Theorem 2.2 and Theorem 2.3 in the case of gluing of vertices 1 and 3 in quiver (3.1).

Case 3.4. Gluing of vertices 1 and $(m+3)$ $(1 \leq m < n)$ in the quiver

$$\begin{tikzcd}
1 \arrow[dr, "\alpha"] & & & & & & \\
& 3 \arrow[r, dash, "\gamma_1"] & \ldots \arrow[r, "\gamma_m"] & (m+3) \arrow[r, "\gamma_{m+1}"] & (m+4) \arrow[r, dash] & \ldots \arrow[r, dash, "\gamma_n"] & (n+3)\,. \\
2 \arrow[ur, dash, "\beta"'] & & & & & &
\end{tikzcd} \tag{3.2}$$

Note that, if the arrow γ_m is directed from vertex $(m+3)$, then even for $m = n$, in the algebra A, we get the following wild subquiver without relations:

$$\begin{tikzcd}
2 \arrow[r, dash, "\beta"] & 3 & \ldots \arrow[l, dash, "\gamma_1"] & (m+3) \arrow[l, "\gamma_m"] \arrow[ll, bend right, "\alpha"'] \,.
\end{tikzcd}$$

If the arrows directed from the vertex $(m+3)$ are absent, then gluing of 1 and $(m+3)$ is inessential. Hence, we can assume that, for $n > m$, the arrow γ_m is directed to vertex $(m+3)$ and the arrow γ_{m+1} is directed from this vertex. The direction of the arrow β does not affect the representation type because this arrow is definitely not contained in the relation and, hence, it is possible to perform a reflection (according to [4]) at point 2 and change its direction. In what follows, we assume that $\beta\colon 2 \to 3$.

As a result, we obtain the quiver

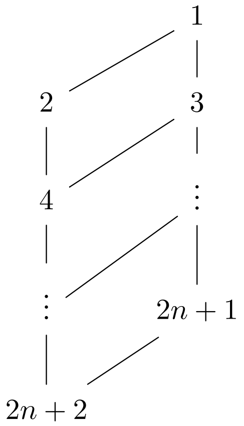
$$\begin{tikzcd}
2 \arrow[r, "\beta"] & 3 \arrow[r, dash, "\gamma_1"'] & \ldots \arrow[r, "\gamma_m"'] & (m+3) \arrow[ll, bend right, "\alpha"'] \arrow[r, "\gamma_{m+1}"] & (m+4) \arrow[r, dash, "\gamma_{m+2}"] & \ldots \arrow[r, dash, "\gamma_n"] & (n+3)
\end{tikzcd}$$

with relation $\alpha\gamma_m = 0$.

As a result of the reduction of the matrices β and γ_i, which form a quiver of the type A_{n+2}, the matrix α is split into $2(n+1)$ horizontal strips and several vertical strips. It follows from the relation $\alpha\gamma_2 = 0$ that the nonzero strips in the matrix α are exhausted by the vertical strips corresponding to the representations of the subquiver

$$\begin{tikzcd}
(m+3) \arrow[r, "\gamma_3"] & (m+4) \arrow[r, dash, "\gamma_4"] & \ldots \arrow[r, dash, "\gamma_n"] & (n+3)
\end{tikzcd}$$

nonzero at the vertex $(m+3)$. The number of these strips is equal to $n - m + 1$. Moreover, their addition is controlled by a linearly ordered set. The addition of horizontal strips is controlled by a partially ordered set S:



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According to [5], the obtained problem is equivalent to the problem of representation of a partially ordered set, i.e., of a cardinal sum S, and a linearly ordered set containing $n-m$ elements. According to [5, 6], this problem and, hence, the algebra A have finite types for $m \leq 3$, $n=m+1$, and are tame for $m=1$, $n=3$ or $m=4$, $n=5$ and wild, otherwise. This proves the second assertion of Theorem 2.1 and the third and fourth assertions of Theorem 2.2. Any additional operations of gluing or blowing up give wild algebras, which proves Theorem 2.3 in the case of gluing of vertices 1 and $(m+3)$ $(m>0)$ in quiver (3.2).

Case 3.5. Blowing up of vertex 3 in quiver (2.6).

As a result, we obtain the quiver

$$\begin{tikzcd}
1 \arrow[d, "\alpha'"'] \arrow[dr, "\alpha''"] & 4 \arrow[d, "\gamma''"] \arrow[dl, "\gamma'"'] \\
3' \arrow[d, "\beta'"'] \arrow[dr, "\delta'"] & 3'' \arrow[d, "\delta''"] \arrow[dl, "\beta''"'] \\
2 & 5
\end{tikzcd}$$

with the following relations: $\beta'\alpha' = \beta''\alpha''$, $\delta'\gamma' = \delta''\gamma''$, $\beta'\gamma' = \beta''\gamma''$, and $\delta'\alpha' = \delta''\alpha''$. In [10], it is proved that this algebra is tame. This gives the sixth assertion of Theorem 2.2 and the fifth assertion of this theorem because quiver (2.5) is a subquiver of quiver (2.6).

Hence, Theorems 2.1 and 2.2 are completely proved. To prove Theorem 2.3, it remains to show that the following operations give a wild algebra:

(1) gluing of the end vertices under the condition that this operation is essential, i.e., the arrow leaves one of the vertices and enters the other vertex;

(2) blowing up of vertex 3 in a quiver of type D_{n+3} or $\widetilde{\mathrm{D}}_{n+3}$ if either $n>1$ or at least three arrows enter this vertex or leave it;

(3) essential gluings with participation of at least one of vertices 1 and 2 and at least one of vertices $1'$ and $2'$ in a quiver of the type $\widetilde{\mathrm{D}}$.

It is easy to see that operations (1) and (2) give a wild subquiver without relations. Operation (3) is split into the cases similar to Cases 3.1–3.4 considered above. It is easy to see that, in all cases, we observe an additional partition of the matrices. This partition transforms the corresponding problems into wild. This completes the proof of Theorem 2.3.

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