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Minimax observability and optimality conditions for noncausal linear differential-algebraic equations

In this talk we discuss optimality conditions for noncausal linear differential-algebraic equations (DAEs)

$$\frac{d}{dt}Fx(t) = C(t)x(t) + f(t), Fx(t_0) = f_0 \tag{1}$$

from the control theory point of view. We present an example of the linear DAE so that corresponding LQ-control problem

$$I(u,z) = \int_{t_0}^{T} ((Q(t)z(t), z(t)) + (R^{-1}u(t), u(t))dt \to \min_{u,z}$$

with DAE constrains

$$\frac{d}{dt}F'z(t) = -C'(t)z(t) + H'(t)u(t), F'z(T) = F'\ell \tag{2}$$

has nontrivial solutions but celebrated Pontryagin maximum principle (PM)

$$\frac{d}{dt}F'z(t) = -C'(t)z(t) + H'(t)R(t)H(t)p(t), F'z(T) = F'\ell,$$
$$\frac{d}{dt}Fx(t) = C(t)x(t) + Q^{-1}(t)z(t), Fx(t_0) = 0$$

describes only the trivial one. In this context we highlite the roots of the described problem and introduce new sufficient conditions generalizing PM for linear noncausal DAEs with variable matrices.

We introduce a new notion of minimax directional observability[1]-[4]: DAE (1) is called obsevable in the minimax sence if

$$\sigma(u) = \sup_{x(\cdot),f(\cdot)\in G} E[(\ell, Fx(T)) - \int_{t_0}^{T} (H'u, x)dt]^2 < \infty$$

provided that G is convex bounded closed set, x denotes the solution of (1) that corresponds to f, u is some measurable vector-function and H is continuous matrix-valued function. We prove that DAE (1) is obsevable in the minimax sence iff (2) has a solution $z(\cdot)$. In particular, regular DAE is observable in the minimax sense for any direction in contrast to the classical observability. We apply the introduced notion to derive the new equations describing the dynamics of the minimax recursive estimator for non-causal DAE. We illustrate our approach with numerical examples.

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[4] Zhuk, S. and Demidenko, S. and Nakonechniy, A., Minimax state observation in linear one-dimentional 2-point boundary value problems, Tavrian Bulletin of Mathematics and Informatics, 2007