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An analogue of Cayley's Theorem for dimonoids

Jean-Louis Loday [1] introduced the notion of a dimonoid. This notion is a standard tool in the theory of Leibniz algebras. Recall that a set D equipped with two associative operations $\prec$ and $\succ$ satisfying the following axioms:

$$(x \prec y) \prec z = x \prec (y \succ z),$$
$$(x \succ y) \prec z = x \succ (y \prec z),$$
$$(x \prec y) \succ z = x \succ (y \succ z)$$

for all $x, y, z \in D$ is called a dimonoid. If the operations of a dimonoid coincide, then the dimonoid becomes a semigroup.

Let X be a nonempty set. We denote by $\Im(X)$ the symmetric semigroup on X. Let $(D, \prec, \succ)$ be an arbitrary dimonoid and let ε be an arbitrary symbol, $\varepsilon \notin D$. For every $a \in D$ we define transformations ρ^a and ρ_a of the set $D \bigcup \{\varepsilon\}$, assuming

$$x\rho^a = \begin{cases} x \succ a, \ x \in D, \\ a, \ x = \varepsilon, \end{cases} \quad x\rho_a = \begin{cases} x \prec a, \ x \in D, \\ a, \ x = \varepsilon \end{cases}$$

for all $x \in D \bigcup \{\varepsilon\}$.

Let $T_r^{\succ}(D) = \{\rho^a | a \in D\}$, $T_r^{\prec}(D) = \{\rho_a | a \in D\}$. It is clear that the sets $T_r^{\succ}(D)$ and $T_r^{\prec}(D)$ are the subsemigroups of the symmetric semigroup $\Im(D \bigcup \{\varepsilon\})$. Define the operations $\prec'$ and $\succ'$ on $T_r^{\succ}(D) \times T_r^{\prec}(D)$ by

$$(\rho^a, \ \rho_b) \prec' (\rho^c, \ \rho_d) = (\rho^a \rho_c, \ \rho_b \rho_d),$$
$$(\rho^a, \ \rho_b) \succ' (\rho^c, \ \rho_d) = (\rho^a \rho^c, \ \rho_b \rho_d)$$

for all $(\rho^a, \ \rho_b), \ (\rho^c, \ \rho_d) \in T_r^{\succ}(D) \times T_r^{\prec}(D)$.

Lemma 1. $(T_r^{\succ}(D) \times T_r^{\prec}(D), \ \prec', \ \succ')$ is a dimonoid.

We denote by $T_r(D)$ this dimonoid.

Lemma 2. A set $\bar{T}_r(D) = \{(\rho^a, \rho_b) \in T_r(D) | a = b\}$ is a subdimonoid of the dimonoid $T_r(D)$.

The following theorem gives an analogue of Cayley's Theorem.

Theorem 3. Every dimonoid $(D, \prec, \succ)$ is isomorphic to the dimonoid $\bar{T}_r(D)$.

If the operations of a dimonoid $(D, \prec, \succ)$ coincide, then from Theorem 3 we obtain Cayley's Theorem [2] for semigroups.

[1] J.-L. Loday, Dialgebras, In: Dialgebras and related operads, Lecture Notes in Math. 1763, Springer, Berlin, 2001, pp.7-66.

[2] A.H.Clifford, G.B.Preston, The algebraic theory of semigroups, vol.1, 2, American Mathematical Society, 1964, 1967.