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An analog of Stampacchia method in the study of solutions of nonlinear fourth-order equations with a strengthened ellipticity

We consider the Dirichlet problem for nonlinear fourth-order equations of the form

$$\sum_{|\alpha|=1,2} (-1)^{|\alpha|} D^\alpha A_\alpha(x, \nabla_2 u) + a|u|^{\sigma-1}u = f \quad \text{in} \quad \Omega,$$

where Ω is a bounded open set of R^n, $f : \Omega \to R$, $a \geq 0$, $\sigma > 1$ and $\nabla_2 u = \{D^\alpha u : |\alpha| = 1, 2\}$. The main structural requirement for the coefficients A_α is the following strengthened ellipticity condition: for a.e. $x \in \Omega$ and every $\xi = \{\xi_\alpha \in R : |\alpha| = 1, 2\}$,

$$\sum_{|\alpha|=1,2} A_\alpha(x, \xi)\, \xi_\alpha \geq c \left\{ \sum_{|\alpha|=1} |\xi_\alpha|^q + \sum_{|\alpha|=2} |\xi_\alpha|^p \right\},$$

where $p \in (1, n/2)$, $q \in (2p, n)$ and $c > 0$.

In the talk we discuss the next questions:

(i) dependence of the integrability of solutions to the given problem on the integrability of the function f in the case where $f \in L^t(\Omega)$ with $t > nq/(nq - n + q)$;

(ii) description of the sets of boundedness for solutions to the given problem in the case where $f \in L^1(\Omega)$.

Similar questions are studied for nonlinear equations of arbitrary even order with strengthened ellipticity and integral functionals with strengthened coercivity.

The results are partially published in [2, 3, 4]. Their proofs are based on the development of Stampacchia method proposed in [1] for second-order equations.

This is a joint talk with A.A. Kovalevsky.

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[3] Voitovich M.V. // Trudy IAMM NAS of Ukraine . — Donetsk, 2006. — **13**.

[4] Voitovich M.V. // Trudy IAMM NAS of Ukraine . — Donetsk, 2007. — **15**.